Questions & Answers

O LEVEL MATHEMATICS

R H Evans

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Questions & Answers

by

R.H. Evans, B.A., B.Sc.

INTRODUCTION

This book is intended to help students about to take examinations at the GCE 'O' level in Mathematics. The questions are drawn from both the University of London and the Associated Examining Board's papers. These Boards have given their kind permission for reproduction of their questions in this text, however we must point out that this in no way implies that the solutions given are the responsibility of either Board. The solutions are the sole responsibility of the author.

The format of the book is specifically structured so that students may read and attempt questions before referring to the suggested answer. In this respect it is a useful self testing program.

The author of this text is a long standing member of the teaching profession specialising in Mathematics and Statistics. Prior to entering the profession many years ago he was an engineer and thus his experience in the applied field is invaluable.

NEIL FULLER
& TONY HINES

4 Ainsdale Close, Bromborough, Merseyside. L63 0EU.

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Question | Page

1. A bank offers two schemes of investment. Scheme A pays tax-free interest of 8%. Scheme B pays interest of 12% on which tax at 30% has to be paid. A man has £1000 to invest. Calculate his income, after tax, under the two different schemes. — 1

2. The graph of $y = x + \frac{4}{x}$ has a minimum point for $x > 0$. — 1

Calculate the coordinates of this minimum point. (You are <u>not</u> required to verify that it is a minimum point.)

3. (i) Solve the equation $x^2 + 4x = 0$. — 2

(ii) Solve the equation $x^2 + 4x + 1 = 0$, giving your answers correct to one decimal place.

4. The vertices of the quadrilateral PQRS have coordinates — 2

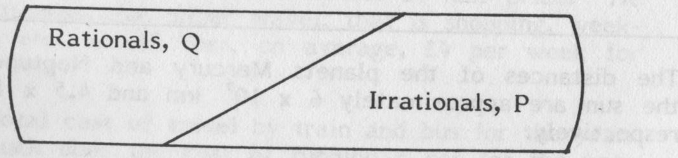
P(0,1), Q(1,3), R(3,5) and S(5,6).

(a) Write down the volumn vectors which represent $\vec{QR}$ and $\vec{PS}$ and state a geometrical relationship between QR and PS.

(b) Write down the column vectors which represent $\vec{PQ}$ and $\vec{RS}$ and show that PQ = RS.

(No credit will be given for constructions or drawings on graph paper.)

5. — 2



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5. (Continued) Page

The Venn diagram shows the set of all real numbers, which are either rational or irrational. 2

(a) Copy the diagram and put into it both the set of integers, I, and the set of natural numbers, N.

(b) For each of the following numbers, state <u>all</u> of the sets, using the letters I, N, P, Q, to which each belongs: (i) π, (ii) 3 1/7, (iii) 4, (iv) -4.

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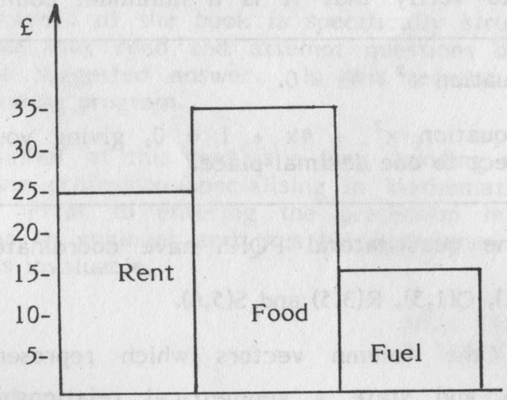
6. 3



The bar chart illustrates the weekly expenditure of a family on rent, food and fuel. Sketch a pie chart to represent this information, marking the size of the angle in each sector.

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7. Make b the subject of the formula 3

$$y = \frac{m(a - b)}{a + b}$$

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8. The distances of the planets Mercury and Neptune from the sun are approximately 6×10^7 km and 4.5×10^9 km respectively. 4

Continued....

8. (Continued) Page

(a) Find, in standard form, the value of 4

$$\frac{\text{distance of Neptune from the sun}}{\text{distance of Mercury from the sun}}$$

(b) Given that the speed of light is 3×10^5 km/s, find, in minutes, the time taken for light to travel from the sun to Neptune.

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9. A man stands on horizontal ground with his feet 50 m from the base of a vertical tower. He observes the angle of elevation of the top of the tower to be 12° and the angle of depression of the base of the tower to be 2°. Find, in metres correct to one decimal place, the height of the tower. 4

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10. In January 1980 a man decided to buy a car for £5000. During the year he travelled 19,000 km at an average petrol consumption of 8.1 litres per 100 km. Petrol cost 29p per litre, insurance cost £105 for the year, servicing charges were £17 and £35 and he had to replace 2 tyres at £16.50 each. At the end of the year he sold the car for 85% of its cost price. Find the total cost of his motoring for the year and calculate, to the nearest 0.1p, the cost per kilometre. 5

If he had not bought the car he would have had to travel to work by bus and train. Assuming that he works for 230 days in the year and every day he buys a return bus ticket for 80p and a return train ticket for £1.30, find the yearly cost of travelling to work by bus and train.

For his holiday he would have to buy 2 adult train tickets at £32 each and two children's tickets at half price. He further estimates that other travel, that is shopping, weekend trips, etc., would cost, on average, £4 per week for a 52 week year.

Find the total cost of travel by train and bus for the year. By how much does the cost of running a car for the year

Continued.....

10. (Continued)

Page

exceed the cost of using bus and train? Express this excess cost as a percentage, to 2 significant figures, of the cost of running the car. 5

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11. In an election, with just 2 candidates, x voters voted for candidate A and 30 voted for candidate B. If a voter is to be picked at random, write down an expression for the probability that a voter will be picked who voted for candidate A. 6

In a second election, with the same 2 candidates, there were 30 more voters altogether but 4 fewer voted for candidate A. If, again, a voter is to be picked at random, write down an expression for the probability that a voter will be picked who voted for candidate A.

Given that the first probability is twice the second probability, form a quadratic equation in x.

Hence find the value of x.

---

12. (a) Show that the point with coordinates (3,4) lies on the curve with equation $y = x^3 - 3x^2 + 4$. Calculate the gradient of the curve at this point. 7

(b) A hydrogen atom consists of an electron and a proton. In appropriate units, the energy E of the atom is given by

$$E = \frac{1}{x^2} - \frac{k}{x} \quad (x \neq 0)$$

where k is a non-zero constant and x is the (variable) distance between the electron and the proton.

Show that E has a turning point when $x = \frac{2}{k}$.

For this value of x, determine the energy of the atom in terms of the constant k. Show that this energy is negative.

Page

13. A function f is defined by $f: x \to 3x - x^2$ for all values of x. 8

(i) Calculate the coordinates of the points where the graph of $y = f(x)$ cuts the x-axis. Make a quick free-hand sketch of the graph.

(ii) Evaluate

$$\int_0^3 f(x)dx.$$

(iii) With reference to the graph of $y = f(x)$, explain briefly why it is possible to have a value of b (where $b > 3$) for which

$$\int_0^b f(x)dx = 0$$

Find this value of b.

(iv) By considering the symmetry of the graph of $y = f(x)$ over the interval $-2 < x < 5$, find the value of a for which

$$\int_a^3 f(x)dx = 0$$

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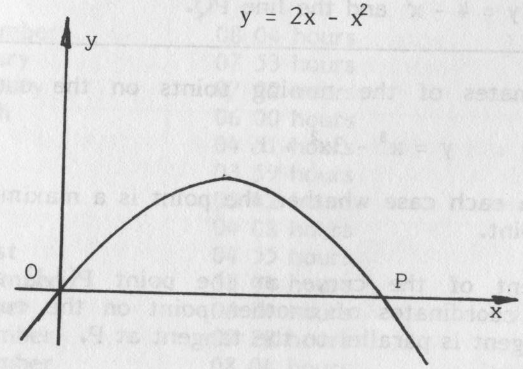
14. (a) The diagram represents part of the curve 9

$$y = 2x - x^2$$



14. (Continued) Page

(i) Write down the value of the x coordinate of P.

(ii) Evaluate the area bounded by the curve and the x-axis.

(b) The equation of a curve is

$$y = 2x^3 - 5x^2 - x.$$

Calculate the acute angle between the x-axis and the tangent to the curve at the point (2,-6).

---

15. Draw the graph of $y = 4 - x^2$ for values of x from $x = -3$ to 3, taking 2 cm to represent 1 unit on each axis. 10

Using the same scales and axes draw the graph of the line $y = x + 2$.

Mark the intersections of the line and the curve as P and Q.

(a) Write down and simplify the equation in x whose solutions are given by the intersections of the curve and the line. From your graphs obtain the solutions of this equation.

(b) Calculate the area completely enclosed between the curve $y = 4 - x^2$ and the line PQ.

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16. Find the coordinates of the turning points on the curve 11

$$y = x^3 - 3x^2 + 1,$$

and determine in each case whether the point is a maximum or a minimum point.

Find the gradient of the curve at the point P(3,1) and hence find the coordinates of another point on the curve at which the tangent is parallel to the tangent at P.

Page

17. The results of an experiment to investigate how a quantity P is related to a quantity W were recorded as follows: 12

P	0.8	1.5	1.8	2.0	2.5
W	19.5	33.5	39.5	43.5	53.5

Plot these points on a graph, taking 4 cm to represent one unit on the P-axis, taken across the squared paper, and 4 cm to represent 10 units on the W-axis.

Show that P and W could be connected by a law of the form

$$W = aP + b,$$

where a and b are constants. Use your graph to estimate values of a and b.

Find also from your graph the value of P when W = 50.

The value of P corresponding to W = 30 is increased by 50%. Find from your graph the value of W corresponding to this increased value of P.

Calculate a likely value for W when P = 10.

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18. The table gives the time of sunrise in London on the 22nd day of each month of the year. 13

December	08 04 hours
January	07 53 hours
February	07 02 hours
March	06 00 hours
April	04 51 hours
May	03 59 hours
June	03 42 hours
July	04 08 hours
August	04 55 hours
September	05 44 hours
October	06 34 hours
November	07 28 hours
December	08 04 hours

Continued....

18. (Continued)

Using a scale of 1 cm to represent 1 month across the page and 2 cm to represent 1 hour after midnight up the page, draw a graph to show how the time of sunrise varies throughout the year. 13

Taking the year to consist of 12 months of 30 days each, it is approximately true that, d days after December 22nd, the sun rises t minutes after midnight, where

$$t = 360 + 124 \cos d^{o}.$$

Using this formula

(i) find, to the nearest minute, the time of sunrise when d = 133,

(ii) find a value of d, correct to the nearest integer, when the sun rises at 0520 hours,

(iii) find the probability that, on a day chosen at random during the period 22nd December to 22nd June, the sun will rise before 0520 hours.

19. The information below gives details of three journeys, all made on the same day, along a motorway which runs from West to East. 14

Means of transport	Starting point	Time of entering motorway	Stopping time	Time of reaching the end of the motorway
Coach A	West end of the motorway	Noon	12.50 pm to 1.05 pm	2.45 pm
Car	East end of the motorway	Noon	-	2.00 pm
Coach B	East end of the motorway	12.30 pm	1.40 pm to 1.50 pm	3.00 pm

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19. (Continued)

Fig. 1 represents a simple map of the routes taken by coach A, the car and coach B.

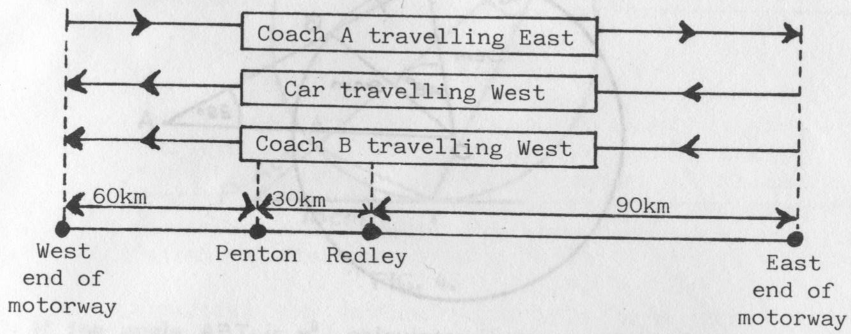

FIG. 1.

Coach A stopped at Penton service station, whilst coach B stopped at Redley service station.

Assuming that the three vehicles moved at steady speeds, draw, with common axes, graphical illustrations of the journeys, taking 2 cm to represent 20 minutes on the time axis and 2 cm to represent 20 km on the distance axis. Mark the time axis as "Number of minutes after noon" and the distance axis as "Number of kilometres from the West end of the motorway".

By marking your graphs clearly, where you take readings, use them to estimate, as accurately as possible:

(a) the time at which coach A and the car were the same distance from the West end of the motorway,

(b) the distance from the East end of the motorway when coaches A and B passed each other,

(c) the distance between the coaches at the time when coach A and the car passed each other,

(d) the time at which the coaches were the greatest distance apart whilst both were travelling on the motorway.

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20. In Fig. 2, O is the centre of a circle of radius 9 cm, ABD is a straight line, the angle BOD = 48^0 and the angle BAO = 28^0. 15

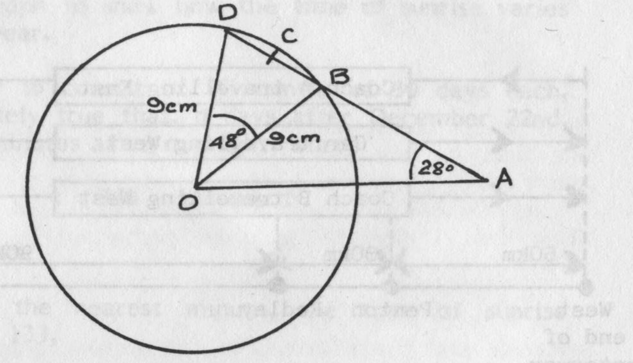

FIG. 2.

(i) Calculate the length of the minor arc BCD.

(ii) Calculate the area of the sector BODC.

(iii) Show that the angle ABO = 114^0.

(iv) Calculate the length of AO.

(v) Calculate the length of AN, where N is the mid-point of BD.

(Take π to be 3.142.)

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21. In Fig. 4, AB is a diameter of the circle, PAB is a straight line and PT is the tangent at T. 16

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21. (Continued)

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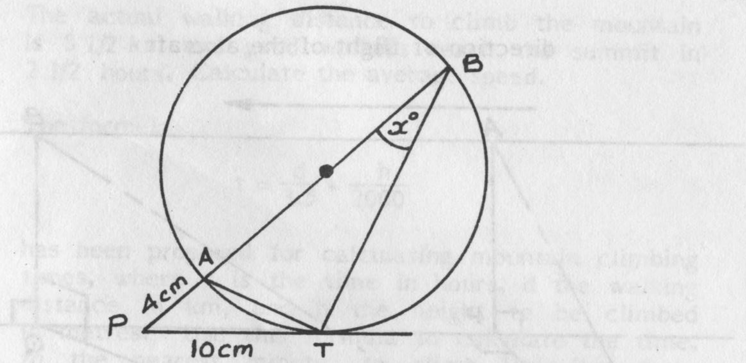

FIG. 4.

If the angle ABT is x^0, calculate, in terms of x, the angles BAT, ATP and APT.

Given that PA = 4 cm and PT = 10 cm calculate

(i) the length PB,

(ii) the radius of the circle,

(iii) the ratio of the area of triangle PAT to the area of the triangle PTB,

(iv) by using similar triangles, or otherwise, the ratio of the length of TA to the length of BT.

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22. In Fig. 5, O is the position of an observer on the horizontal plane OPQ. The observer is watching an aircraft which is flying due east at a constant speed of 400 km/h and at a constant height of 2000 m. 17

When the aircraft is at A, it is due north of O and its angle of elevation from O is 29^0.

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22. (Continued)	Page
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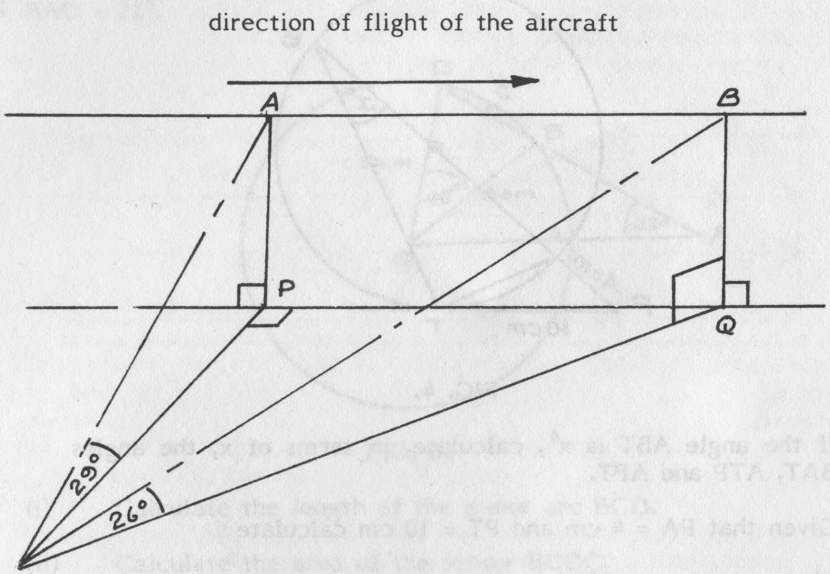

FIG. 5.

Calculate the distance OP.
Later, when the aircraft is at B, its angle of elevation from O is 26^0. Calculate the bearing of the aircraft from O at this instant.
Find the distance AB and hence deduce the time, in seconds to the nearest second, between the two observations.

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23. The summit of the mountain Helvellyn is approximately 1000 m above sea level and the village church gate at Patterdale is 180 m above sea level. The summit is 6 km in a straight line from the church gate. 18

(a) Calculate, to the nearest degree, the angle of elevation of the summit from the church gate.

Continued....

23. (Continued) Page

(b) The actual walking distance to climb the mountain is 8 1/2 km and good walkers reach the summit in 2 1/2 hours. Calculate the average speed.

(c) The formula

$$t = \frac{d}{3.5} + \frac{h}{2000}$$

has been proposed for calculating mountain climbing times, where t is the time in hours, d the walking distance in km, and h the height to be climbed in metres. Use this formula to calculate the time, to the nearest minute, to climb Helvellyn from Patterdale church gate.

(d) Rearrange the formula to express h in terms of the time and the walking distance.

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24. (a) In a parallelogram ABCD, AB = 8 cm, BC = 6 cm and the angle ABC = $117^0 17'$. Calculate the length of the diagonal AC and the size of the angle BAC. 19

(b) A ship steams 7 km East from a position P to a position T. From T the ship changes course to 060^0 and travels in this direction for 10 km to a position X. From X the ship changes course again and travels 15 km South to Y. From Y the ship returns to the position P.

(i) Draw a sketch to illustrate the above information, marking the positions P, T, X and Y.

(ii) Calculate the bearing of the position P from the position Y.

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25. Two lightships A and B are situated at sea 30 miles apart, both on a bearing of 254^0 from a point P on land. The lightship B is 28 miles from P. 20

Continued....

25. (Continued) Page

A ship X is steaming in a direction of 344^0 along a line equidistant from A and B, so that it will pass between them.

Find, to the nearest tenth of a degree, the bearing of X from P, when X is 28 miles from B and before X has reached AB.

Find also the distance, to the nearest mile, of X from P at this time.

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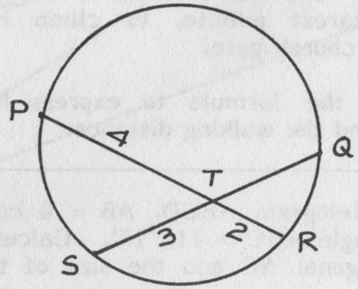
26. 21



In the diagram, T is the point of intersection of the chords PR and SQ of a circle. PT = 4 cm, TR = 2 cm and TS = 3 cm.

(a) Prove that the length of TQ is 2 2/3 cm.

(b) Prove that ΔPTS is similar to ΔQTR.

(c) Given that the area of ΔPTS is 3 cm^2, find the area of ΔQTR.

(d) Find the value of the ratio

$$\frac{\text{area of } \Delta PTQ}{\text{area of } \Delta RTS}$$

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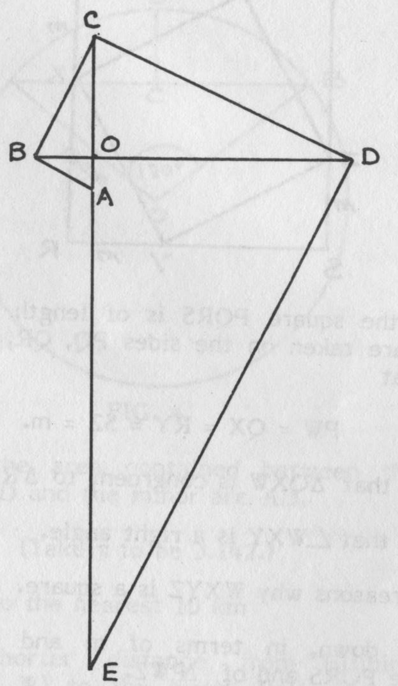
27. 22



The four triangles AOB, BOC, COD and DOE are all similar, and OA = 1 cm, OB = 2 cm.

(a) Find, in cm^2, the area of the whole figure.

(b) Find the length, in cm correct to one decimal place, of BE.

(c) Prove that BC is parallel to ED.

(d) Find, to the nearest degree, the size of the angle BED.

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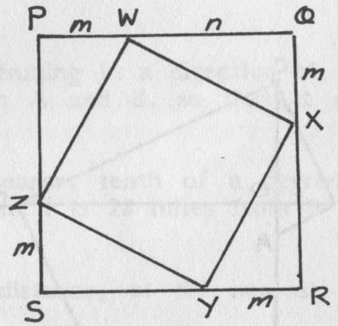
28. 

23

The side of the square PQRS is of length m + n. Points W, X, Y, Z are taken on the sides PQ, QR, RS, SP respectively such that

$$PW = QX = RY = SZ = m.$$

(a) Prove that ΔQXW is congruent to ΔRYX.

(b) Prove that $\angle WXY$ is a right angle.

(c) Give reasons why WXYZ is a square.

(d) Write down, in terms of m and n, the areas of square PQRS and of ΔPWZ.

By considering the areas of the squares and the triangles, verify that $WX^2 = m^2 + n^2$.

(e) Given that WY = 4m, calculate the value of the ratio n : m.

---

29. (a) In Fig. 4, the circle, centre O, has a radius of 10cm and AB is a chord of the circle with the angle AOB = 120^0. The mid points of the chord AB and the minor arc AB are C and D respectively.

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29. (Continued)

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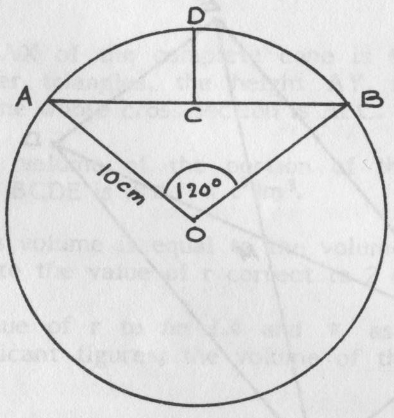

FIG. 4.

Calculate the area contained between the straight lines AC, CD and the minor arc AD.

(Take π to be 3.142.)

(b) Calculate, to the nearest 10 km

(i) the shorter distance from Birmingham (53^0 N, 2^0 W) to the North Pole, measured along the meridian through Birmingham,

(ii) the radius of the circle of latitude 53^0 N.

(iii) the shorter distance along this circle of latitude, from Birmingham to Amsterdam (53^0 N, 3^0 E).

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30. In Fig. 3, ABC is a triangle in which AB = 5 cm, BC = 8 cm and the angle ABC = $97^0 54'$. In the rectangle ACDE, AE = 24 cm and the diagonals meet at M. 25

Calculate

(a) the length of AC,

(b) the size of angle ACB,

(c) the length of AM,

(d) the size of angle DME.

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30. (Continued) Page

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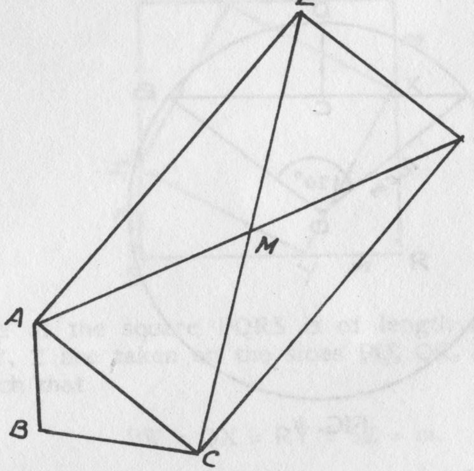

FIG. 3.

31. 26

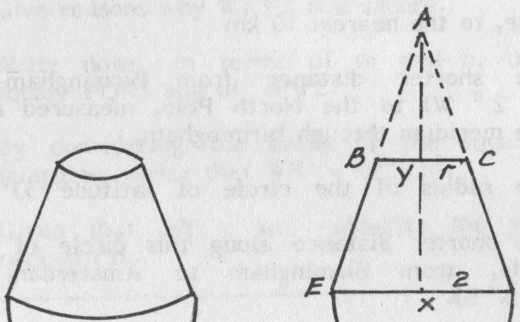

Figure a shows a space capsule which consists of a portion of a cone whose parallel plane ends are circles of radii 2 metres and r metres, joined to a hemisphere of radius 2 metres. In Figure b, ACDEB is the cross-section of the complete cone of which the portion BCDE is the cross-section of the upper portion of the capsule. Given that

Continued....

31. (Continued) Page

the height of AX of the complete cone is 6 metres, find, by using similar triangles, the height AY, in terms of r, of the small cone whose cross-section is ABC. 26

Show that the volume of the portion of the cone whose cross-section is BCDE is $(8\pi - \pi r^3) m^3$.

Given that this volume is equal to the volume of the hemisphere, calculate the value of r correct to 2 decimal places.

Taking the value of r to be 1.4 and π as 3.14, find, in m^3 to 3 significant figures, the volume of the whole space capsule.

32. A closed cylindrical can, of base radius r cm and height h cm, is to be constructed to hold 400 cm^3. Write down an expression for h in terms of r and show that the total area, A cm^2, of sheet metal required to make the can is given by 27

$$A = 2\pi r^2 + \frac{800}{r}$$

Find $\frac{dA}{dr}$ and hence, taking π as 3.14, find, to 2 significant figures, the value of r which makes A a minimum.

For this value of r find the value of h:r.

33. A student has a total of 126 marks in x tests. In the next two tests he has 9 marks and 8 marks respectively. Find, in terms of x, his average number of marks per test for 28

(i) the first x tests,
(ii) the (x + 2) tests.

If his average for the first x tests was one greater than his average for the (x + 2) tests, use the results of (i) and (ii) to form an equation and, hence, find the value of x.

Continued....

33. (Continued)

Page

Another student has an average of 13.5 marks for the first $(x + 1)$ tests, but his mark on the last test gave him a final average of 14 marks for the $(x + 2)$ tests. What was his mark on the last test? 28

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34. (a) Add together the two fractions 29

$$\frac{2}{x-5} \text{ and } \frac{4}{3-x},$$

and simplify your answer.

(b) Solve the equation

$$\frac{2x-14}{8x-15-x^2} = 1,$$

giving your answers correct to one decimal place.

(c) Sketch the graph of

$$y = x^2 - 6x + 1.$$

Show clearly on your graph the coordinates of the points where the graph cuts the x-axis.

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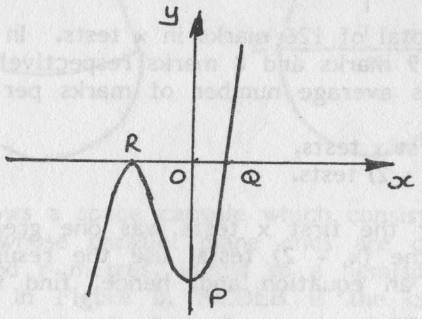
35. 30



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35. (Continued)

Page

The diagram shows the graph of 30

$$y = x^3 + 3x^2 - 4$$

The graph cuts the y-axis at P, cuts the x-axis at Q and touches the x-axis at R.

(a) Find the coordinates of P.

(b) Given that Q is the point (1,0), find the coordinates of R.

(c) Find the coordinates of the two points on the curve where the gradient is 9.

(d) If T is the point (0, -9), find the area of ΔQTR.

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36. It is given that $p = a - b$ and $q = bp + p^2$, 31

(i) find the values of p and q, when $a = 2$ and $b = -3$.

(ii) By substituting $(a - b)$ for p in the expression $(bp + p^2)$, and simplifying the result, show that a formula for q, in terms of a and b, is $q = a(a - b)$.

(iii) When $q = 1/2$ and $b = -1/3$ show that $q = a(a - b)$ can be expressed as $6a^2 + 2a - 3 = 0$.

(iv) Solve the equation $6a^2 + 2a \quad 3 = 0$, giving each answer correct to two decimal places.

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37. (a) Solve the equation $5x^2 - 13x - 7 = 0$ giving your answer to two decimal places. 32

(b) Express $\frac{m-12}{(m-3)(m+3)} + \frac{3}{2(m-3)}$ as a single fraction in its lowest terms.

Continued....

37. (Continued)

Page

(c) Given that 32

$$a = \frac{b-c}{b+c}$$

(i) calculate a when b = 17 and c = 8.

(ii) express c in terms of a and b.

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38. A car and a lorry travel in the same direction along a motorway. The car travels at a constant speed of x km/h and the speed of the lorry, which is also constant, is 30 km/h slower than that of the car. 33

Write down, in terms of x, expressions for

(a) the speed, in km/h, of the lorry,

(b) the time taken, in hours, by the car to travel 20km,

(c) the time taken, in hours, by the lorry to travel 20km.

The car takes 6 minutes less than the lorry to cover the distance of 20 km. Write down an equation which x must satisfy and show that it simplifies to

$$x^2 - 30x \quad 6000 = 0$$

Solve this equation, giving your solutions correct to one decimal place, and hence find the speed of the lorry.

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39. (a) In an examination, the lowest and highest marks were 36 and 61 respectively. In order to change any mark, y, into a new mark, N, the formula N = 4(y - 36) was used. Calculate the lowest and highest mark on the new scale and the mark which remained unchanged. 34

(b) By setting out each step of working clearly, show that the equation $4x^2 + 12x - 11 = 0$ results from simplifying $(2x + 3)^2 = 20$.

Continued....

39. (Continued)

Page

Hence, or otherwise, solve the equation $4x^2 + 12x - 11 = 0$, giving your answers correct to two decimal places. 34

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40. (a) Solve the equations 35

$$p - 2q = 3,$$
$$pq = 2.$$

(b) The total cost, C pence, of manufacturing a cubical block of side x centimetres is represented by the formula $C = ax + bx^2$. Given that the cost of manufacturing a block of side 2 cm is 4 pence and a block of side 4 cm is 14 pence, form two equations in a and b and hence find the values of a and b.

Also, find the cost of manufacturing one of these blocks of side 6 cm.

---

41. (i) On squared paper using a scale of 1 cm to represent 1 unit, draw axes to show values of x from -10 to +10 and values of y from 0 to +10. Draw and label the triangle ABC where A, B and C are the points (5,0), (10,0) and (10,5) respectively. 36

(ii) The images of the triangle ABC under transformations represented by the matrices **P** and **Q** are triangles $A_1B_1C_1$ and $A_2B_2C_2$ respectively. Given that

$$\mathbf{P} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad \text{and} \quad \mathbf{Q} = -\frac{1}{5}\begin{pmatrix} 3 & 4 \\ -4 & 3 \end{pmatrix}$$

find, draw and label $A_1B_1C_1$ and $A_2B_2C_2$ on your diagram, taking care to label each vertex correctly.

Describe fully the single transformation represented by **Q**. (Any value that you need in your description should be taken from your diagram.)

Continued....

41. (Continued) Page

(iii) Find the inverse matrix P^{-1} and hence determine the matrix **X** such that **X**P = Q. 36

Describe fully the single transformation represented by X.

---

42. In this question use a scale of 2 cm to represent 1 unit on each axis, taking values of x from -1 to +7 and values of y from -5 to +5. 37

Draw on graph paper ΔABC, where A, B, C are respectively the points (2,2), (6,2) and (6,4).

The transformation E is an enlargement about the point (4,0) with scale factor -1/2. The transformation R is a rotation through 180^0 about the point (3,0).

Construct the images of ΔABC under the transformations E, R, ER and RE marking your diagram carefully to distinguish the 4 images.

Describe in words the single transformation T such that TRE = ER.

Write down the matrix of the transformation S such that SRE = I, where I is the identity transformation, and describe in words the transformation S.

---

43. The points A(1,1), B(5,1) and C(3,2) are joined to form ΔABC. 38

(a) On graph paper, using 1 cm to a unit, and putting the origin in the lower left corner of the paper, draw ΔABC.

(b) Calculate the coordinates of the vertices of $\Delta A'B'C'$, which is formed by transforming ΔABC using the matrix

$$\begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}$$

Continued....

43. (Continued) Page

(c) Draw $\Delta A'B'C'$ on the same graph. 38

(d) Calculate the coordinates of the vertices of $\Delta A''B''C''$ which is formed by transforming $\Delta A'B'C'$ using the matrix

$$\begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix}$$

(e) Draw $\Delta A''B''C''$ on the same graph and state the scale factor of the enlargement from ΔABC to $\Delta A''B''C''$.

(f) State the ratios of the areas of the three triangles.

---

44. A binary operation * is defined on the set 39

$$S = \{ (r,\theta) : r \geqslant 0,\ 0 \leqslant \theta < 360 \}$$

such that

$$(a,b)*(c,d) = (ac, b \oplus d)$$

where $\oplus$ represents addition modulo 360.

(i) Evaluate (2, 120)*(3, 300).

(ii) Evaluate (a,b)*(1,0). What does this suggest about (1,0)?

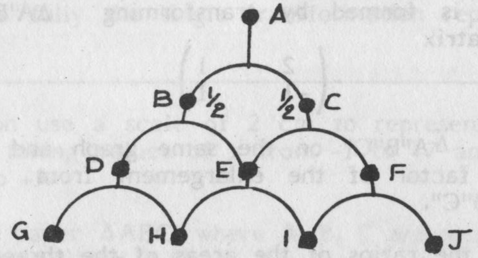
(iii) Evaluate $(p,q)*(\frac{1}{p}, 360 - q)$ and hence write down the inverse of (2/3, 170).

(iv) Find r and θ if $(r,\theta)*(3,70) = (6,30)$.

(v) Find r and all possible values of θ if

$$[(r,\theta)*(r,\theta)*(r,\theta)] = (8,0).$$

45. A motorist turns left with probability 1/2 and turns right with probability 1/2 whenever he comes to a T-junction. The motorist sets off from town A in the following road system so that the probability he will go to B is 1/2 and the probability he will go to C is 1/2, as shown.



(a) Make a copy of the diagram, and mark on it the probabilities that he will reach D, E, F, G, H, I and J.

(b) Explain why the sum of the last four probabilities in (a) should be 1.

(c) Another motorist turns left with probability q and turns right with probability p, so that on the same road system, we would write p by B and q by C.

State the probabilities that this second motorist reaches (i) D, (ii) E, (iii) F.

(d) Work out $(p + q)^2$ and show that this is the same as the sum of your three answers to (c).

46. Six functions are defined by

$$i : x \to x, \qquad f : x \to 1 - x, \qquad g : x \to \frac{1}{x},$$

$$h : x \to 1 - \frac{1}{x}, \qquad j : x \to \frac{1}{1 - x}, \qquad k : x \to \frac{x}{x - 1}.$$

(a) Copy and complete the combination table below. (Note that for fg, f appears in the left column and g in the top row and fg = h.)

Continued....

46. (Continued)

	i	f	g	h	j	k
i	i	f	g	h	j	k
f	f	i	h	g		
g	g	j	i	k		
h	h	k	f	j		
j	j	g	k	i		
k	k	h	j	f		

(b) State the identity function.

(c) State the inverse functions for (i) j, (ii) k.

(d) Use the table to simplify

(i) (fg)h, (ii) f(gh).

47. In the regular hexagon OPQRST, OP = p and OT = t. 42

Express PT in terms of p and t and show that

(a) PS = 2t,

(b) OS = p + 2t.

Given that PX = 2/3 PT, show that X lies on OS and find the value of $\dfrac{OX}{XS}$.

48. Sets are defined as follows: 43

$\mathcal{E}$ = cars in a certain car park
R = red cars
F = cars not made in Great Britain
A = cars with automatic transmission
H = cars with a rear door ("hatchbacks")
M = cars with engines of more than 2 litres capacity
S = cars with sunshine roofs

Continued....

48. (Continued)

Page

Write sentences, not using set language, to express the following statements: 43

(a) $F' \cap H = \emptyset$

(b) $H \cap R = H$

(c) $A \cup M = A$

Express the following statements in set language:

(d) None of the red cars has both automatic transmission and a sunshine roof.

(e) Only cars made in Great Britain have engines of more than 2 litres.

(f) All the cars not made in Great Britain have sunshine roofs.

---

49. A council is replacing its fleet of buses. It has been agreed that there must be at least 50 new buses and that the number of double decker buses must not be less than half the number of single decker buses. 44

The council buys x single decker and y double decker buses.

(i) Write down two inequalities (other than $x \geqslant 0$, $y \geqslant 0$) which satisfy the above conditions. Using a scale of 2 cm to represent 10 buses, illustrate these inequalities on squared paper. Shade the unwanted regions.

The seating capacity for a single decker is 40, for a double decker it is 60. The council decides to buy enough buses to have a total seating capacity of exactly 2400.

(ii) Write down and simplify the equation which satisfies this condition. On your diagram draw the graph of the line which has this equation.

Continued....

49. (Continued)

Page

(iii) Mark with small circles the possible ordered pairs (x,y) which satisfy all the conditions in (i) and (ii). 44

A double decker bus requires two crewmen, a single decker only one.

(iv) Find the minimum number of men required to crew the new fleet of buses.

---

50. Taking a scale of 2 cm to represent one unit on the x-axis and 1 cm to represent one unit on the y-axis and using the same axes for both graphs, draw, for $-2 \leqslant x \leqslant 3$, the graphs of the functions 45

$$f : x \to x - 2,$$
$$g : x \to x^2.$$

Copy and complete the statements

$$gf : x \to \text{..........}, \text{ and } f^{-1} : x \to \text{.........}$$

Using the same scales and axes draw the graphs of the functions $gf(x)$ and $f^{-1}(x)$.

Find from your graphs the values of x for which

(a) $gf(x) = f(x)$,

(b) $f^{-1}(x) = g(x)$.

---

(1) The man invests £1000, his income :-

(a) Scheme A	(b) Scheme B
8% of £1000	12% of £1000
$= \frac{8}{100} \times £1000$	$= \frac{12}{100} \times £1000$
$= £80$ income	$= £120$

After tax, he receives

$100\% - 30\% = 70\%$ of £120

$= \frac{70}{100} \times £120$

$= £84$ income

(2) The graph has the equation : $y = x + 4/x$

ie $y = x + 4x^{-1}$

$\frac{dy}{dx} = 1 - 4x^{-2}$

$\therefore \frac{dy}{dx} = 1 - \frac{4}{x^2}$

For min. value, $\frac{dy}{dx} = 0$

$1 - \frac{4}{x^2} = 0$

$x^2 = 4$

$\therefore x = \pm 2$

It is given that, for min. point, $x > 0$

$\therefore$ min. point has x-coordinate :- $x = 2$.

putting $x = 2$ in $y = x + 4/x$

$y = 2 + 4/2$

$\therefore y = 4$

$\therefore$ min. point has y-coordinate :- $y = 4$

3) (i) $x^2 + 4x = 0$

$\therefore x(x+4) = 0$

$\therefore x = 0$ or $x + 4 = 0$

$\therefore x = 0$ or -4

(ii) $x^2 + 4x + 1 = 0$

using formula :-

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

where $a = 1$, $b = 4$, $c = 1$

$x = \frac{-4 \pm \sqrt{16 - 4}}{2}$

$x = \frac{-4 \pm \sqrt{12}}{2}$

$x = \frac{-4 \pm 2\sqrt{3}}{2}$

$x = -2 \pm \sqrt{3}$

$x = -0.3$ or -3.7

4) If O is the origin, the position vectors of :-

$P = \vec{OP} = \binom{0}{1}$, $Q = \vec{OQ} = \binom{1}{3}$, $R = \vec{OR} = \binom{3}{5}$, $S = \vec{OS} = \binom{5}{6}$

(a) $\vec{QR} = \vec{QO} + \vec{OR} = \binom{-1}{-3} + \binom{3}{5} = \binom{2}{2}$

$\vec{PS} = \vec{PO} + \vec{OS} = \binom{0}{-1} + \binom{5}{6} = \binom{5}{5}$

$\vec{QR}$ and $\vec{PS}$ are parallel

(b) $\vec{PQ} = \vec{PO} + \vec{OQ} = \binom{0}{-1} + \binom{1}{3} = \binom{1}{2}$

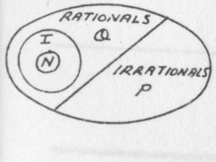
$\vec{RS} = \vec{RO} + \vec{OS} = \binom{-3}{-5} + \binom{5}{6} = \binom{2}{1}$

$PQ = \sqrt{1^2 + 2^2} = \sqrt{5}$

$RS = \sqrt{2^2 + 1^2} = \sqrt{5}$

$\therefore PQ = RS$ (as required)

(a)



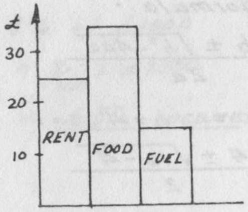
(b) (i) $\pi \in P$ ie, π is irrational.

(ii) $3\frac{1}{7} \in Q$ ie, $3\frac{1}{7}$ is rational.

(iii) $4 \in Q$, $4 \in I$, $4 \in N$ ie, 4 is rational, an integer and natural.

(iv) $-4 \in Q$, $-4 \in I$ ie, -4 is rational and an integer

(6)



Expenditure on :-

Rent = £25
Food = £35
Fuel = £15

$\left.\right\}$ TOTAL = £75

For Pie Chart, 360° represents £75

$\therefore$ 24° " £5

5 × 24° = 120° represents £25 expenditure on Rent

7 × 24° = 168° " £35 " " Food

3 × 24 = 72° " £15 " " Fuel

Pie Chart :-

---

(7)

$$y = \frac{m(a-b)}{a+b}$$

$$\therefore\ y(a+b) = m(a-b)$$

$$\therefore\ ay + by = am - bm$$

$$\therefore\ bm + by = am - ay$$

$$\therefore\ b(m+y) = a(m-y)$$

$$\therefore\ b = \frac{a(m-y)}{(m+y)}$$

(a) $\dfrac{\text{Distance of Neptune from the Sun}}{\text{Distance of Mercury from the Sun}} = \dfrac{4.5 \times 10^9}{6 \times 10^7}$

$= 0.75 \times 10^2$

$= 7.5 \times 10$

(b) Time taken $= \dfrac{\text{Distance}}{\text{Speed}}$

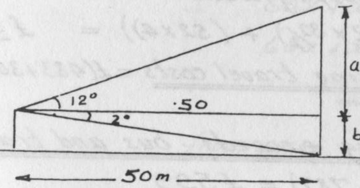
$= \dfrac{4.5 \times 10^9}{3 \times 10^5}$

$= 1.5 \times 10^4$ secs

$= \dfrac{1.5 \times 10^4}{60}$ mins

$= 0.025 \times 10^4$

$=$ 250 minutes ie 2.5×10^2 minutes



$\tan 12° = \dfrac{a}{50}$ $\tan 2° = \dfrac{b}{50}$

$\therefore a = 50 \tan 12°$, $b = 50 \tan 2°$

height of tower $= a + b$

$= 50 \tan 12° + 50 \tan 2°$

$= 50(\tan 12° + \tan 2°)$

$= 12.37$

$= 12.4$ metres

---

(10) Purchase price of car = £5000

Petrol consumption $= \frac{19\,000}{100} \times 8.1 = 1539$ litres

Petrol Cost $= £\frac{1539 \times 29}{100} = £446.31$

Insurance, servicing, tyres $\Big\} = £(105 + 17 + 35 + 2(16.50)) = £190$

Total Expenditure $= £(5000 + 446.31 + 190) = £5636.31$

Income (Sale of car) $= 85\% \times £5000 = £4250$

Total motoring costs $= £(5636.31 - 4250) = £1386.31$

Cost per km. $= \frac{138631}{19000} = 7.3$ pence (to nearest 0.1p)

Daily bus and train cost $= £(0.80 + 1.30) = £2{-}10$

Yearly bus and train cost $= £2{-}10 \times 230 = £483$

Yearly holiday, shopping, weekend costs :-

$£((2 \times 32) + (2 \times 32\tfrac{1}{2}) + (52 \times 4)) = £304$

Total non-motoring travel costs $= £(483 + 304) = £787$

Cost of motoring exceeds bus and train costs by $£(1386 - 787) = £599$

% Excess cost $= \frac{599}{1386} \times 100\% = 43\%$ (to 2 Sig figs)

x voters voted for candidate A

30 " " " B

$\therefore (30 + x)$ = Total number of votes cast.

P(voter casted vote for A) $= \frac{x}{(30+x)}$

In second election, $(x-4)$ voted for candidate A.

$(30+x) + 30 = 60 + x$ = Total votes cast

$\therefore P$(voter casted vote for A) $= \frac{x-4}{60+x}$

From the given information, $\frac{x}{30+x} = \frac{2(x-4)}{60+x}$

$\therefore x(60+x) = 2(x-4)(30+x)$

$\therefore 60x + x^2 = 2(30x - 120 + x^2 - 4x)$

$\therefore 60x + x^2 = 2x^2 + 52x - 240$

$\therefore 0 = x^2 - 8x - 240$

from which $(x - 20)(x + 12) = 0$

$\therefore x = 20$ or -12

$\therefore$ rejecting $x = -12$,

we have:- $x = 20$

(12) (a) We have, $y = x^3 - 3x^2 + 4$

$$\frac{dy}{dx} = 3x^2 - 6x$$

$$\therefore \underline{\text{gradient at } (3,4)} = 3(3)^2 - 6(3) = 27 - 18 = \underline{9}$$

(b) We have, $E = \frac{1}{x^2} - \frac{k}{x}$ --- (1)

$$\therefore E = x^{-2} - kx^{-1}$$

$$\therefore \frac{dE}{dx} = -\frac{2}{x^3} + \frac{k}{x^2}$$

for a turning point, $\frac{dE}{dx} = 0$

ie, $-\frac{2}{x^3} + \frac{k}{x^2} = 0$

ie $\frac{k}{x^2} = \frac{2}{x^3}$

ie $\underline{x = \frac{2}{k}}$, as required

putting $x = \frac{2}{k}$ in (1)

$$E = \frac{1}{\left(\frac{4}{k^2}\right)} - \frac{k}{\left(\frac{2}{k}\right)}$$

$$E = \frac{k^2}{4} - \frac{k^2}{2}$$

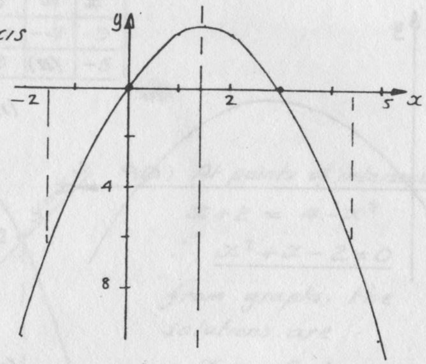
$$\underline{E = -\frac{k^2}{4}}$$

3) We have $f : x \to 3x - x^2$, for all x

(i) The graph cuts the x-axis when $3x - x^2 = 0$

ie, $x(3-x) = 0$

ie, $\underline{x = 3, 0}$



(ii) Let $I = \int_0^3 f(x)\,dx$

$$= \int_0^3 (3x - x^2)\,dx$$

$$= \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^3$$

$$= \left(\frac{27}{2} - \frac{27}{3}\right) - (0 - 0)$$

$$= \frac{27}{6}$$

$$= \underline{4.5}$$

(iii) If $b > 3$, the area between the curve, the x-axis, the line $x = 3$ and the line $x = b$ will be below the x-axis and, therefore, will be negative. If b is such that this area will be equal to the area above the x-axis, we have the case $\int_0^b f(x)\,dx = 0$

in this case,

$$\int_0^b (3x - x^2)\,dx = 0$$

$$\therefore \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]^b = 0$$

$$\left(\frac{3b^2}{2} - \frac{b^3}{3}\right) = 0$$

$$\therefore 9b^2 - 2b^3 = 0$$

$$\therefore b^2(9 - 2b) = 0$$

from which, $\underline{b = \frac{9}{2} = 4.5}$

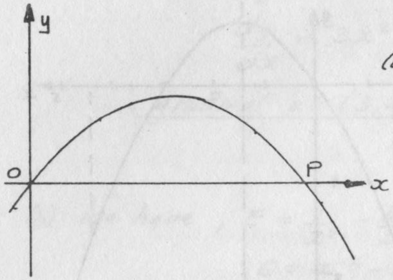
(iv) The axis of symmetry is $x = 1.5$

The integral is zero between the limits $x = 0$ and $x = 4.5$

By symmetry, the integral is also zero between the limits $x = -1.5$ and $x = 3$

ie, $\underline{a = -1.5}$

(14)



(a) $y = 2x - x^2$

(i) P has x-coordinate > 0 and y-coordinate = 0

$\therefore$ at P, $0 = 2x - x^2$

$\therefore 0 = (2-x)x$

$\therefore x = 0, 2$

$\therefore$ P is the point (2,0)

(ii) Area $= \int_0^2 (2x - x^2)\,dx$

$= \left[x^2 - \frac{x^3}{3}\right]_0^2$

$= \left(4 - \frac{8}{3}\right) - (0 - 0)$

$= \frac{4}{3}$ square units

(b) $y = 2x^3 - 5x^2 - x$

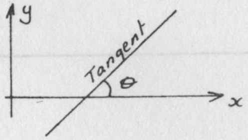
$\frac{dy}{dx} = 6x^2 - 10x - 1$

$\frac{dy}{dx}$ = gradient of curve at any point on the curve.

= gradient of the tangent to the curve at the point concerned.

when $x = 2$, $\frac{dy}{dx} = 24 - 20 - 1 = 3$

$\therefore$ at point $(2, -6)$ gradient of tangent $= 3$



gradient of tangent = $\tan\theta$

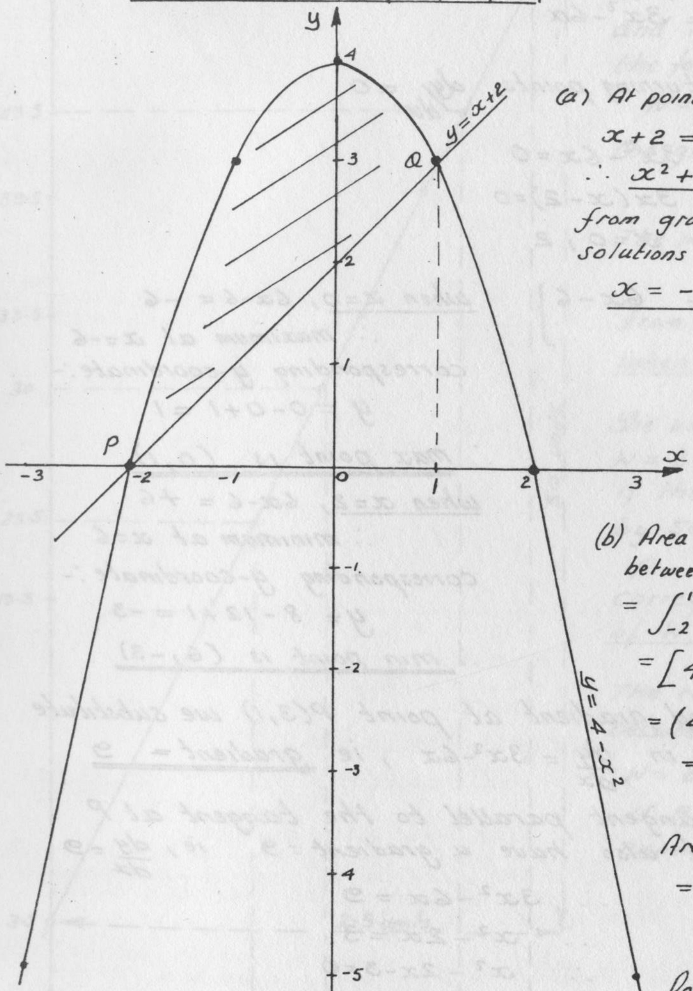
$\therefore \tan\theta = 3$

$\therefore \theta = 71°34'\ (71.57°)$

where θ = acute angle between tangent and x-[axis]

(15)

x	-3	-2	-1	0	1	2	3
4	4	4	4	4	4	4	4
$-x^2$	-9	-4	-1	0	-1	-4	-9
y	-5	0	3	4	3	0	-5



(a) At points of intersection,

$x + 2 = 4 - x^2$

$\therefore x^2 + x - 2 = 0$

from graphs, the solutions are :-

$x = -2, 1$

(b) Area under curve between $x = -2$ and $x = 1$

$= \int_{-2}^{1} (4 - x^2)\,dx$

$= \left[4x - \frac{x^3}{3}\right]_{-2}^{1}$

$= (4 - 1/3) - (-8 + \frac{8}{3})$

$= 4 - 1/3 + 8 - 8/3$

$= 9$ sq. units

Area of triangle

$= 1/2 \times 3 \times 3$

$= \frac{9}{2}$ sq units

Required (shaded) area $= 9 - 9/2$

$= \frac{9}{2}$ sq units

(16) $y = x^3 - 3x^2 + 1$

$$\frac{dy}{dx} = 3x^2 - 6x$$

for turning points, $dy/dx = 0$

$\therefore \quad 3x^2 - 6x = 0$

$\therefore \quad 3x(x-2) = 0$

$\therefore \quad x = 0, 2$

$\frac{d^2y}{dx^2} = 6x - 6 \}$ when $x=0$, $6x-6 = -6$

$\therefore$ maximum at $x = -6$

corresponding y-coordinate:-

$y = 0 - 0 + 1 = 1$

$\therefore$ max point is $(0,1)$

when $x=2$, $6x-6 = +6$

$\therefore$ minimum at $x = 6$

corresponding y-coordinate:-

$y = 8 - 12 + 1 = -3$

$\therefore$ min point is $(6, -3)$

To find gradient at point $P(3,1)$ we substitute $x = 3$ in $\frac{dy}{dx} = 3x^2 - 6x$, ie gradient $= 9$

A tangent parallel to the tangent at P must also have a gradient $= 9$ ie, $\frac{dy}{dx} = 9$

$\therefore \quad 3x^2 - 6x = 9$

$\therefore \quad x^2 - 2x = 3$

$\therefore \quad x^2 - 2x - 3 = 0$

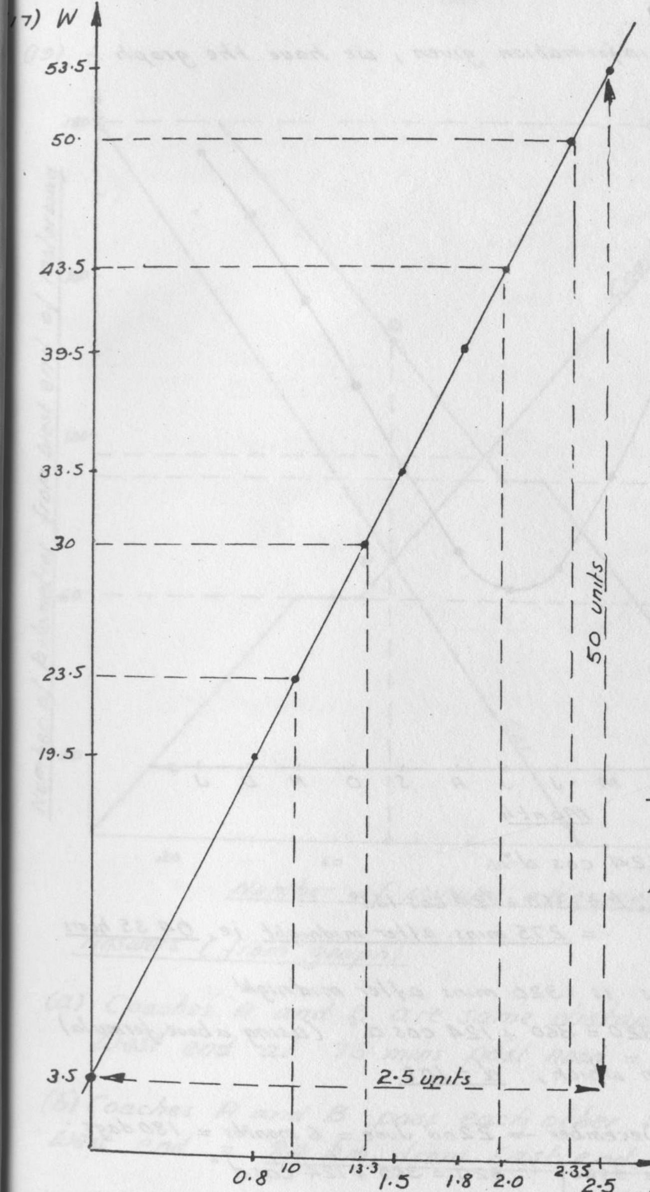
$\therefore \quad (x-3)(x+1) = 0$

$\therefore \quad x = 3, -1.$

for $x = 3$, $y = 27 - 27 + 1$

$\therefore$ required point is $(3,1)$

(17)



The resulting graph is a straight line and is therefore of the form

$W = aP + b$

The gradient $a = \frac{50}{2.5}$

$\therefore \underline{a = 20}$

The intercept on y-axis $\underline{b = 3.5}$

from the graph:-

when $W = 50$, $P = 2.35$

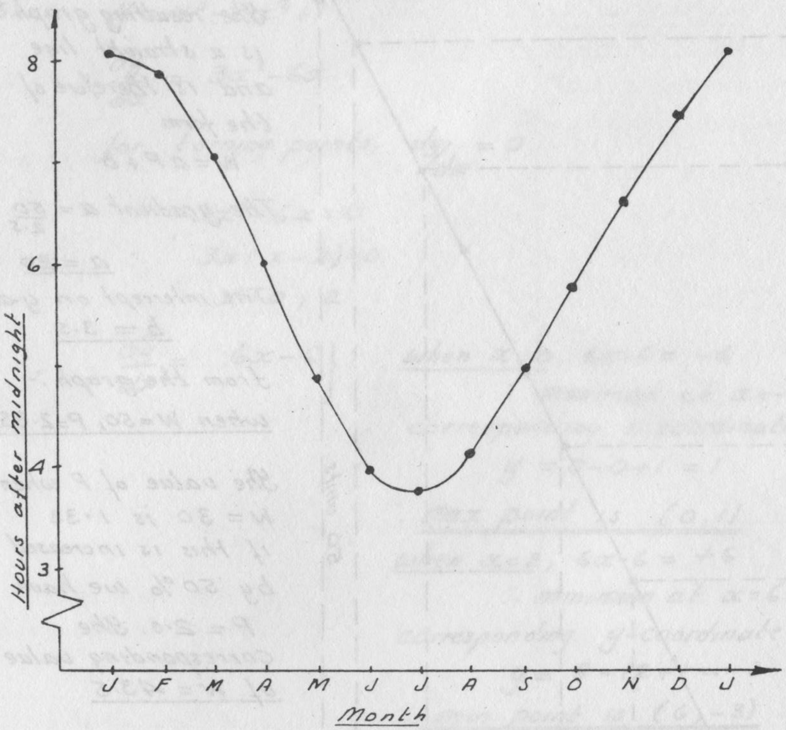
The value of P when $W = 30$ is 1.33 if this is increased by 50% we have $P = 2.0$. The corresponding value of $W = 43.5$

The relationship between P and a:-

$W = 20P + 3.5$

If $P = 10$, we expect $W = 235$

(18) Using the information given, we have the graph :



$t = 360 + 124 \cos d^\circ$

(i) if $d = 133$, $t = 360 + 124 \cos 133^\circ$

$= \underline{275 \text{ mins. after midnight}}$ ie, $\underline{04\ 35 \text{ hours}}$

(ii) 05 20 hours is 320 mins after midnight

$\therefore \quad 320 = 360 + 124 \cos d^\circ$ (using above formula)

from which, $\underline{d = 108}$

(iii) 22nd December → 22nd June = 6 months = 180 days

for time 0520, $t = 320$ $\Big\}$ $\therefore 320 = 360 + 124 \cos d^\circ$

$\therefore \cos d^\circ = \dfrac{320 - 360}{124}$

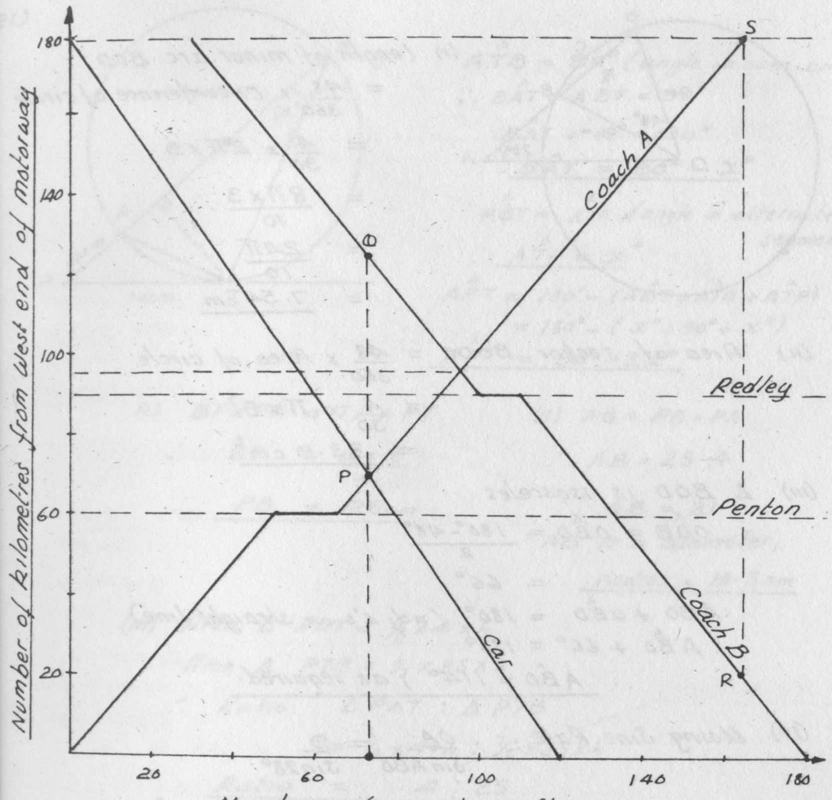
$\therefore \quad d = 108 \cdot 8$ and $251 \cdot 2$

$\therefore$ for given range of 180 days, sun rises before 0520 on $180 - 108 \cdot 8 = 71$ days

$\therefore$ required probability $= \dfrac{71}{180} = \underline{0 \cdot 39}$

U.O.L Jan 83 - B2 - 12

(19)



Answers (from graph)

(a) Coaches A and B are same distance from West end at 73 mins past noon = $\underline{13\ 13}$

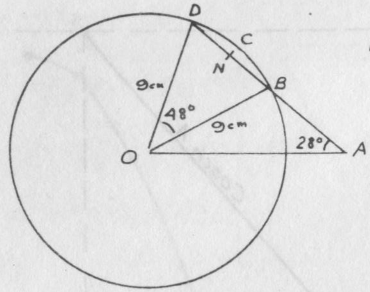
(b) Coaches A and B pass each other 96 km from West end = $\underline{84 \text{ km from East end}}$

(c) Distance between the coaches (PQ) = $\underline{54 \text{ km}}$

(d) Greatest distance apart (RS) at time 165 mins past noon = $\underline{14\ 45}$

A.E.B Nov 83 - B - A5

(20.)



(i) length of minor arc BCD

$= \frac{48}{360} \times$ circumference of circle

$= \frac{4}{30} \times 2\pi \times 9$

$= \frac{8\pi \times 3}{10}$

$= \frac{24\pi}{10}$

$= \underline{7.54\text{ cm}}$

(ii) Area of sector BODC $= \frac{48}{360} \times$ Area of circle

$= \frac{4}{30} \times \pi \times 9^2$

$= \underline{33.9\text{ cm}^2}$

(iii) Δ BOD is isosceles

$\therefore O\hat{D}B = O\hat{B}D = \frac{180° - 48°}{2}$

$= 66°$

$A\hat{B}O + O\hat{B}D = 180°$ (adj $\angle$'s on straight line)

$\therefore A\hat{B}O + 66° = 180°$

$\therefore \underline{A\hat{B}O = 114°}$, as required.

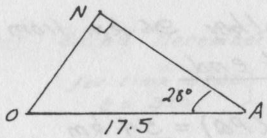
(iv) Using Sine Rule :- $\frac{OA}{\sin A\hat{B}O} = \frac{9}{\sin 28°}$

$\therefore OA = \frac{9}{\sin 28°} \times \sin 114°$

$\therefore \underline{OA = 17.5\text{ cm}}$

(v) $O\hat{N}A = 90°$ (N is mid point of base DB of isosceles Δ ODB)

$\therefore \Delta$ ONA is right angled

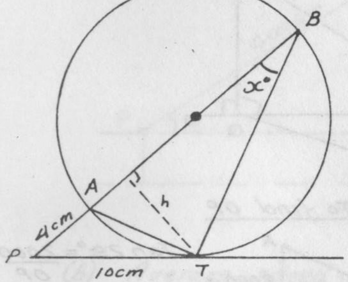


$\cos N\hat{A}O = \frac{AN}{OA}$

$\therefore \cos 28° = \frac{AN}{17.5}$

$\therefore \underline{AN = 15.5\text{cm}}$

(21.)



$A\hat{T}B = 90°$ (angle in semi-circle)

$\therefore B\hat{A}T + A\hat{B}T = 90°$

$\therefore B\hat{A}T + x° = 90°$

$\therefore \underline{B\hat{A}T = 90° - x°}$

$A\hat{B}T = A\hat{T}P$ (angle in alternate segment)

$\therefore \underline{A\hat{T}P = x°}$

$A\hat{P}T = 180° - (A\hat{B}T + A\hat{T}B + A\hat{T}P)$

$= 180° - (x° + 90° + x°)$

$\therefore \underline{A\hat{P}T = 90° - 2x°}$

(i) $BP \times PA = TP \times PT$

$\therefore BP = \frac{10 \times 10}{4}$

$\therefore \underline{PB = 25\text{cm}}$

(ii) $AB = PB - PA$

$\therefore AB = 25 - 4$

$\therefore AB = 21$

AB is a diameter,

$\therefore \underline{\text{radius} = 10.5\text{ cm}}$

(iii) Area Δ PAT $= \frac{1}{2} \times 4 \times h$

Area Δ PTB $= \frac{1}{2} \times 25 \times h$

$\therefore$ Ratio $\Delta PAT : \Delta PTB$

$= \frac{1}{2} \times 4 \times h : \frac{1}{2} \times 25 \times h$

$\underline{\text{Ratio} = 4 : 25}$

(iv) In Δ's, TPA, BPT

$A\hat{T}P = T\hat{B}P = x°$

$T\hat{P}A = B\hat{P}T$ (common)

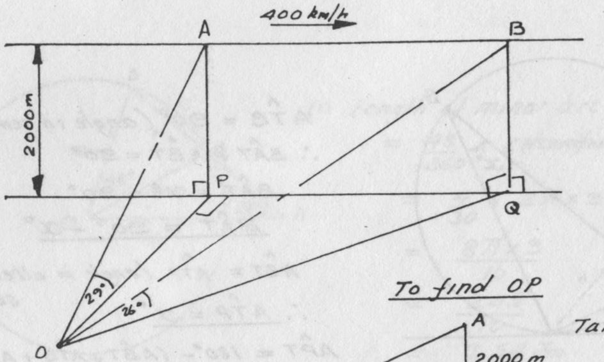
$\therefore \Delta$'s TPA, BPT similar, in ratio TP:BP

ie 10 : 25

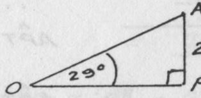
ie 2 : 5

$\therefore \underline{\text{Ratio } TA : BT = 2:5}$

(22)



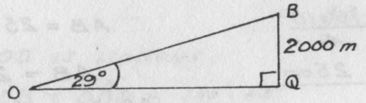
To find OP



$\tan 29° = \dfrac{2000}{OP}$

$\therefore OP = \dfrac{2000}{\tan 29°}$

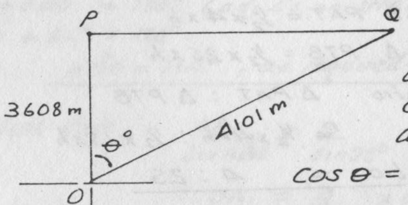
$OP = 3608\,m$



$\tan 26° = \dfrac{2000}{OQ}$

$\therefore OQ = \dfrac{2000}{\tan 26°}$

$= 4101\,m$



The bearing of the aircraft from O is given by angle θ

$\cos\theta = \dfrac{3608}{4101}$

$\therefore \theta = 28°23'\ (28\cdot38)$

$AB = PQ$

By Pythagoras Theorem, $AB^2 = 4101^2 - 3608^2$

$\therefore AB = 1949\,m$

Time to travel AB at 400 km/h

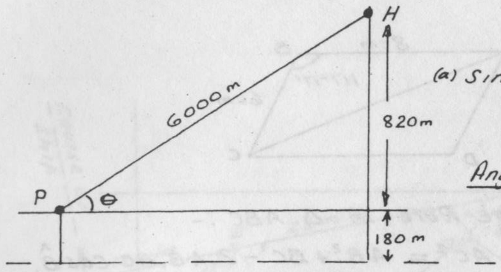
$= \dfrac{1949}{400 \times 1000}$ hours

$= \dfrac{1949 \times 60 \times 60}{400 \times 1000}$

$= 17.5$ secs

$= 18$ secs (to nearest sec)

(23)



(a) $\sin\theta = \dfrac{820}{6000}$

$\therefore \theta = 8°$ (nearest degree)

Angle of elevation $= 8°$

(b) Average speed $= \dfrac{\text{total distance}}{\text{total time}} = \dfrac{8\frac{1}{2}}{2\frac{1}{2}}$ km/hour.

$= 3\cdot4$ km/hour

(c) $t = \dfrac{d}{3.5} + \dfrac{h}{2000}$

$d = 8\frac{1}{2},\ h = 820$

$\therefore t = \dfrac{8\frac{1}{2}}{3.5} + \dfrac{820}{2000}$

$= 2\cdot838$ hours

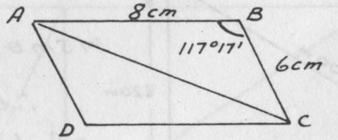
$=$ 2 hours 50 minutes (to nearest minute)

(d) $t = \dfrac{d}{3.5} + \dfrac{h}{2000}$

$t - \dfrac{d}{3.5} = \dfrac{h}{2000}$

$2000\left\{t - \dfrac{d}{3.5}\right\} = h$

(24) (a)



Using Cosine Rule in Δ ABC :-

$$AC^2 = AB^2 + BC^2 - 2.AB.BC \cos \hat{B}$$

$$AC^2 = 64 + 36 - 2.8.6 \cos 117°17'$$

$$AC^2 = 144$$

$$\therefore AC = 12.$$

Using Sine Rule in Δ ABC.

$$\frac{\sin \hat{A}}{BC} = \frac{\sin \hat{B}}{AC}$$

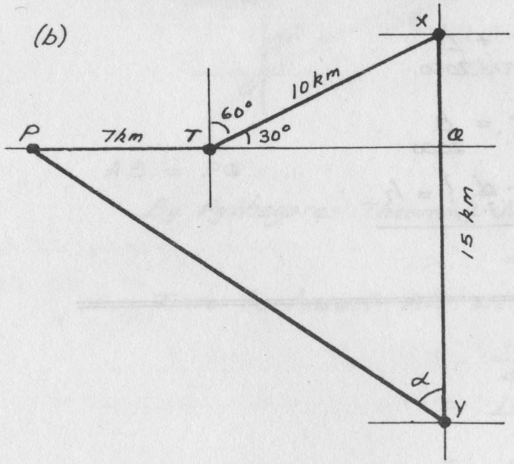
$$\therefore \sin \hat{A} = \frac{BC . \sin \hat{B}}{AC}$$

$$= \frac{6 \times \sin 117°17'}{12}$$

$$\sin \hat{A} = 0.4443$$

$$\therefore B\hat{A}C = 26°23'$$

(b)



$XQ = 10 \sin 30° = 5$

$\therefore QY = 15 - 5 = 10km$

$TQ = 10 \cos 30° = 8.66$

$\therefore PQ = 7 + 8.66 = 15.66 \, km$

In Δ PQY,

$$\tan \alpha = \frac{PQ}{QY} = \frac{15.66}{10}$$

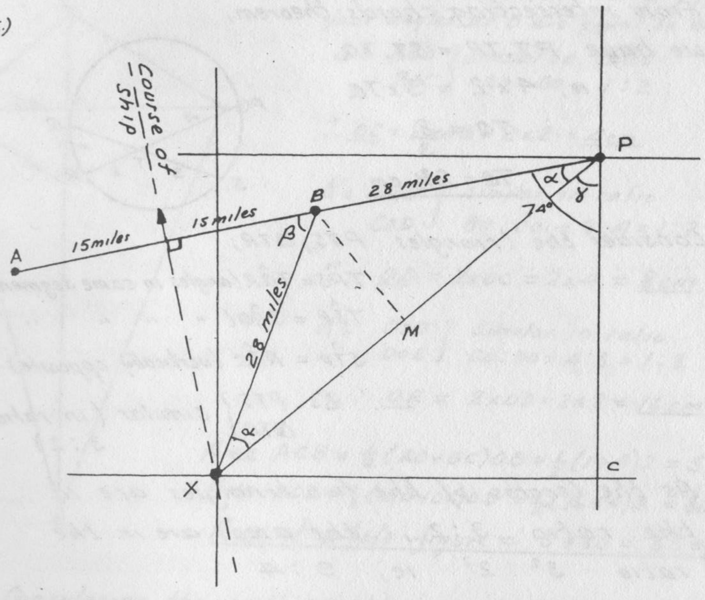
$$\therefore \alpha = 56°26'$$

$\therefore$ Bearing of P from Y

$= 360° - 57°26'$

$= 302°34'$

(25)



Referring to the diagram :-

$B\hat{P}C = 254° - 180°$

$\therefore B\hat{P}C = 74°$

$\cos \beta = \frac{15}{28}$, $\therefore \beta = 57.6°$

$2\alpha = \beta$ (ext. angle of triangle = sum of opp int. angles)

$\therefore 2\alpha = 57.6°$, $\therefore \alpha = 28.8°$

$\gamma = 74° - \alpha$

$\gamma = 74° - 28.8° = 45.2°$

The bearing of X from P $= 180° + 45.2° = 225.2°$

$XP = 2 \times MP$

$= 2 \times 28 \cos \alpha$

$= 2 \times 28 \cos 28.8°$

$= 49$

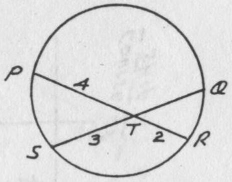
Distance of X from P = 49 miles

(26) (a) From intersecting chords theorem, we have $PT.TR = ST.TQ$

ie, $4 \times 2 = 3 \times TQ$

$\therefore TQ = \frac{8}{3}$

$\therefore \underline{TQ = 2\tfrac{2}{3} \text{ cm.}}$



(b) Consider the triangles PTS, QTR;

$T\hat{P}S = T\hat{Q}R$ (angles in same segment)

$T\hat{S}P = T\hat{R}Q$ (" " " ")

$S\hat{T}P = R\hat{T}Q$ (vertically opposite)

$\therefore$ Δ's $\left.\begin{matrix} PTS \\ QTR \end{matrix}\right\}$ similar (in ratio 3 : 2)

(c) As the lengths of the two triangles are in the ratio 3 : 2; the areas are in the ratio $3^2 : 2^2$ ie, 9 : 4

Thus the area of $\Delta\, QTR = \frac{4}{9} \times$ area of $\Delta\, PTS$

$= \frac{4}{9} \times 3$

$= \underline{\frac{4}{3} \text{ cm}^2}$

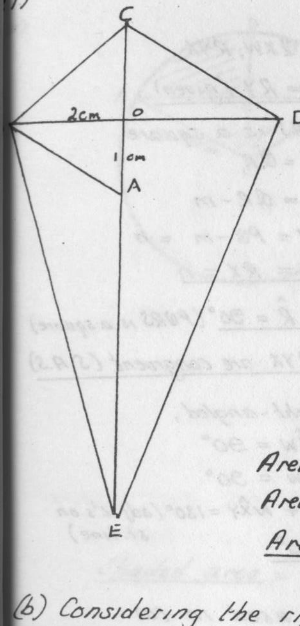
(d) By following an argument similar to that of part (b), it can be shown that:-

Δ's $\left.\begin{matrix} PTQ \\ STR \end{matrix}\right\}$ similar (in ratio 4 : 3)

$$\therefore \frac{\text{Area of } \Delta PTQ}{\text{Area of } \Delta STR} = \frac{4^2}{3^2} = \frac{16}{9}$$

$$\therefore \frac{\text{Area of } \Delta PTQ}{\text{Area of } \Delta RTS} = \frac{16}{9}$$

7)



(a) As Δ's AOB, BOC, COD, DOE similar, the four angles at O are equal to 90°

Δ's $\left.\begin{matrix} AOB \\ BOC \end{matrix}\right\}$ Similar in ratio $AO : BO = 1 : 2$

$\therefore \underline{OC} = 2 \times OB = 2 \times 2 = \underline{4 cm}$

Δ's $\left.\begin{matrix} BOC \\ COD \end{matrix}\right\}$ Similar in ratio $BO : CO = 2 : 4 = 1 : 2$

$\therefore \underline{OD} = 2 \times OC = 2 \times 4 = \underline{8 cm}$

Δ's $\left.\begin{matrix} COD \\ DOE \end{matrix}\right\}$ Similar in ratio $CO : DO = 4 : 8 = 1 : 2$

$\therefore \underline{OE} = 2 \times OD = 2 \times 8 = \underline{16 cm}$

Area $ACB = \frac{1}{2}(AO + OC)OB = \frac{1}{2}(1+4)2 = 5$ sq units

Area $ECD = \frac{1}{2}(EO + OC)OD = \frac{1}{2}(16+4)8 = 80$ sq units

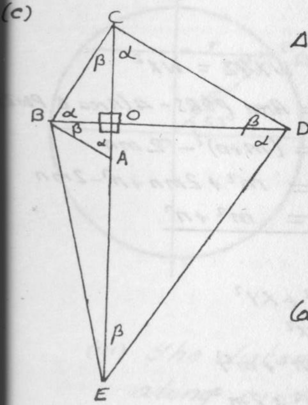
<u>Area of whole figure = 5 + 80 = 85 sq units</u>

(b) Considering the right angled triangle OBE and using Pythagoras, $BE = \sqrt{OE^2 + OB^2}$

$= \sqrt{16^2 + 2^2}$

$\therefore \underline{BE = 16 \cdot 1 \text{ cm}}$

(c)



Δ's AOB, BOC, COD, DOE similar

let α = the equal angles $O\hat{A}B, O\hat{B}C, O\hat{C}D, O\hat{D}E$

and β = " " " $A\hat{B}O, B\hat{C}O, C\hat{D}O, D\hat{E}O$

Now $(\alpha + \beta) = 180° - 90° = 90°$

$\therefore$ BC is parallel to ED

$(B\hat{C}D + E\hat{D}C = 180°)$

(d) $\operatorname{Tan} B\hat{E}O = \frac{OB}{OE} = \frac{2}{16}$

$\therefore B\hat{E}O = 7 \cdot 13°$

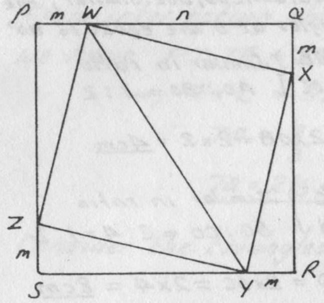
$\operatorname{Tan} D\hat{E}O = \frac{OD}{OE} = \frac{8}{16}$

$\therefore D\hat{E}O = 26 \cdot 57°$

$B\hat{E}D = B\hat{E}O + D\hat{E}O$

$= \underline{34° \text{ to nearest degree.}}$

(28)



(a) In Δ's QXW, RYX

$QX = RY$ (given)

now PQRS is a square

$\therefore PQ = QR$

$RX = QR - m$

$QW = PQ - m = n$

$\therefore QW = RX = n$

$\hat{Q} = \hat{R} = 90^\circ$ (PQRS is a square)

$\therefore$ Δ's QXW, RYX are congruent (S.A.S)

(b) $\Delta QXW \equiv \Delta RYX$

$\therefore Q\hat{W}X = R\hat{X}Y$

and $Q\hat{X}W = R\hat{Y}X$

as Δ QXW is right-angled,

$Q\hat{W}X + Q\hat{X}W = 90^\circ$

$\therefore R\hat{X}Y + Q\hat{X}W = 90^\circ$

$Q\hat{X}W + R\hat{X}Y + W\hat{X}Y = 180^\circ$ (adj. ∠'s on st. line)

$\therefore W\hat{X}Y = 90^\circ$

(c) By an argument similar to that used in (a) it can be proved that the 4 triangles are congruent

$\therefore WX = XY = YZ = ZW$

and. $W\hat{X}Y = 90^\circ$

$\therefore$ WXYZ is a square

(d) Area $PQRS = (m+n)^2$

Area $\Delta PWZ = \frac{1}{2} mn$

Area of $WXYZ = WX^2$

$\therefore WX^2 = $ Area PQRS − 4(Area Δ PW

$= (m+n)^2 - 2mn$

$= m^2 + 2mn + n^2 - 2mn$

$\therefore WX^2 = m^2 + n^2$

(e) $WY = 4m$

now $WY^2 = WX^2 + XY^2$

$\therefore WY^2 = 2WX^2$

$\therefore (4m)^2 = 2(n^2 + m^2)$

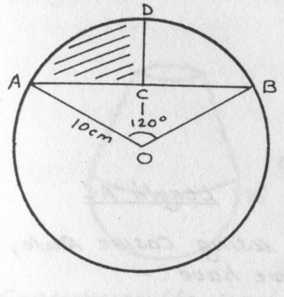
$\therefore 16m^2 = 2n^2 + 2m^2$

$\therefore 7m^2 = n^2$

$\therefore \frac{m}{n} = \frac{1}{\sqrt{7}}$

$m : n = 1 : \sqrt{7}$

(a)



The shaded area, ACD, is required

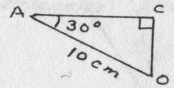
Area of Sector OAD

$= \frac{60^\circ}{360^\circ} \times \pi r^2$

$= \frac{1}{6} \times 3.142 \times 10^2$

$= 52.36 \text{ cm}^2$

Consider Δ ACO,



$OC = 10 \sin 30^\circ$

$\therefore OC = 5\text{cm}$

$AC = 10 \cos 30^\circ$

$AC = 8.66 \text{ cm}$

Area $\Delta ACO = \frac{1}{2} \times AC \times OC$

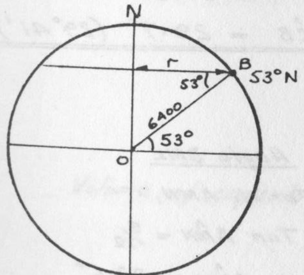
$= 21.65 \text{ cm}^2$

Shaded area = Area Sector OAD − Area Δ ACO

$= 52.36 - 21.65$

$= 30.7 \text{ cm}^2$

(b)



(i) Distance from Birmingham to N. Pole is equal to length of arc BN

$BN = \frac{(90^\circ - 53^\circ)}{360} \times 2 \times 3.142 \times 6400$

$= 4130$ km (to nearest 10 km)

(ii) Radius $r = 6400 \times \cos 53^\circ$

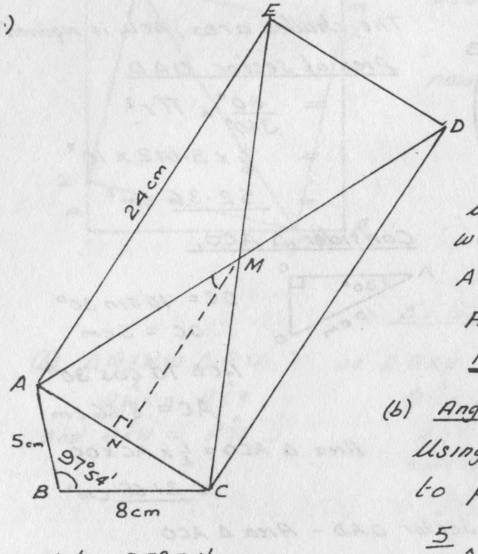
Radius of circle of latitude 53°N $= 3850$ km (to nearest 10 km)

(iii) The distance from Birmingham to Amsterdam along circle of latitude = length of arc which subtends $(3 - -2)^\circ = 5^\circ$ at centre

$= \frac{5}{360} \times 2 \times 3.142 \times 3850$

$= 340$ km (to nearest 10km)

(30.)



(a) Length AC

Using Cosine Rule, we have:-

$AC^2 = 5^2 + 8^2 - 2 \times 5 \times 8 \cos 97{\cdot}8$

$AC^2 = 100$

$AC = 10\,cm$

(b) Angle ACB

Using Sine Rule and answer to part (a), we have:-

$$\frac{5}{\sin A\hat{C}B} = \frac{10}{\sin 97.8^\circ}$$

$$\therefore \sin A\hat{C}B = \frac{5 \sin 97.8^\circ}{10}$$

$$= 0{\cdot}495$$

$$A\hat{C}B = 29{\cdot}7^\circ \ (29^\circ 41')$$

Note $97^\circ 54'$

$= 97{\cdot}9^\circ$

(c) Length AM

Using Pythagoras:-

$AM^2 = AN^2 + MN^2$

$\therefore AM^2 = 5^2 + 12^2$

from which,

$AM = 13\,cm$

(d) Angle DME

For Δ AMN,

$\tan A\hat{M}N = 5/12$

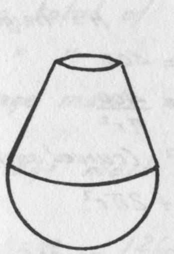
$\therefore A\hat{M}N = 22{\cdot}6^\circ$

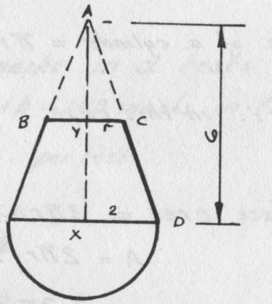
$\therefore A\hat{M}C = 2 \times 22{\cdot}6^\circ$

$= 45{\cdot}2^\circ$

$D\hat{M}E = A\hat{M}C = 45{\cdot}2^\circ$

(31)





Considering the similar triangles AYC, AXD;

$$\frac{AY}{YC} = \frac{AX}{XD}$$

from which, $AY = \frac{6r}{2} = 3r$

Volume of cone AED $= \frac{1}{3}\pi \times 2^2 \times 6 = 8\pi\ m^3$

Volume of cone ABC $= \frac{1}{3}\pi \times r^2 . 3r = \pi r^3 m^3$ } Using "$V(cone) = \frac{1}{3}\pi . r^2 h$"

$\therefore$ Volume of frustum BCDE $= (8\pi - \pi r^3)\ m^3$, as required

"Volume of hemisphere $= \frac{1}{2} \times \frac{4}{3}\pi r^3$"

in this case, $V(hemisphere) = \frac{2}{3} \times \pi \times 2^3 = \frac{16\pi}{3}\ m^3$

thus, we have $8\pi - \pi r^3 = \frac{16\pi}{3}$

$\therefore 8\pi - \frac{16\pi}{3} = \pi r^3$

$\therefore r^3 = \frac{8}{3}$

$\therefore r = \sqrt[3]{\frac{8}{3}} = 1.39$ (to 2 dec places)

Volume of capsule = Vol of hemisphere + Vol of frustum

$\doteq \left(\frac{16\pi}{3} + 8\pi - \pi r^3\right) m^3$

$= \pi\left(\frac{16}{3} + 8 - r^3\right) m^3$

taking $\pi =$ and $r = 1.4$

Volume of capsule $= 3.14\left(\frac{16}{3} + 8 - 1.4^3\right)$

$= 33{\cdot}2\ m^3$ (to 3 sig. figs)

(Note:- 2 x hemisphere volume does not give this answer. This is because r is taken to be 1.4)

(32) Volume of a cylinder $= \pi r^2 h$ where r = base radius (cm)
h = height. (cm)

$\therefore$ in this case :- $\pi r^2 h = 400$

$$\therefore h = \frac{400}{\pi r^2}$$

Surface area $= 2\pi r h + 2\pi r^2$ (curved surface + 2 ends)

$$\therefore A = 2\pi r\left(\frac{400}{\pi r^2}\right) + 2\pi r^2$$

$$= 2\pi r^2 + \frac{800}{r} \text{, as required}$$

$$A = 2\pi r^2 + 800 r^{-1}$$

$$\therefore \frac{dA}{dr} = 4\pi r - \frac{800}{r^2}$$

For min A, $\frac{dA}{dr} = 0$

$$\therefore 4\pi r - \frac{800}{r^2} = 0$$

$$\therefore 4\pi r = \frac{800}{r^2}$$

$$\therefore r^3 = \frac{200}{\pi}$$

$$\therefore \underline{r = 4\cdot 0 \text{ (2 sig. figs)}}$$

for this value of r,

$$h = \frac{400}{\pi \times 4^2}$$

$$\therefore h : r = \frac{400}{\pi \times 16} : 4$$

$$= 400 : 64\pi$$

$$= \underline{25 : 4\pi} \text{ (this answer probably acceptable)}$$

$$= 25 : 12{\cdot}56$$

$$= \underline{\underline{2 : 1}}$$

U.O.L Jan. 83 B2 - 13

(33) He has a total of 126 marks in x tests
and " " " $126 + 9 + 8 = 143$ marks in $(x+2)$ tests

(I) Average marks $= \frac{126}{x}$ per test

(II) Average marks $= \frac{143}{x+2}$ " "

we have $\frac{126}{x} - \frac{143}{x+2} = 1$

$$\therefore 126(x+2) - 143x = x(x+2)$$

$$\therefore 126x + 252 - 143x = x^2 + 2x$$

$$\therefore x^2 + 19x - 252 = 0$$

$$(x - 9)(x + 28) = 0$$

$$x = 9, -28$$

$\underline{x = 9}$ is the only appropriate solution.

For the other student.

Average of $13{\cdot}5$ for $(x+1)$ tests = total of $13{\cdot}5(x+1)$
Average of $14{\cdot}0$ for $(x+2)$ tests = " " $14(x+2)$

$\therefore$ Mark on his last test $= 14(x+2) - 13{\cdot}5(x+1)$

$$= 14x + 28 - 13{\cdot}5x - 13{\cdot}5$$

$$= 0{\cdot}5x + 14{\cdot}5$$

$$= 4{\cdot}5 + 14{\cdot}5$$

$$= \underline{19}$$

A.E.B June 1981 - B - A4

(34) (a) $\dfrac{2}{x-5} + \dfrac{4}{3-x}$

$= \dfrac{2(3-x)+4(x-5)}{(x-5)(3-x)}$

$= \dfrac{6-2x+4x-20}{(x-5)(3-x)}$

$= \dfrac{2x-14}{(x-5)(3-x)}$

$= \dfrac{2(x-7)}{(x-5)(3-x)}$

(b) $\dfrac{2x-14}{8x-15-x^2} = 1$

$\therefore\ 2x-14 = 8x-15-x^2$

$\therefore\ x^2-6x+1=0$

$\therefore\ x = \dfrac{6 \pm \sqrt{36-4}}{2}$ (by formula)

$\therefore\ x = \dfrac{6 \pm \sqrt{32}}{2}$

$\therefore\ x = \dfrac{6 \pm 4\sqrt{2}}{2}$

$\therefore\ x = 3 \pm 2\sqrt{2}$

$\therefore\ \underline{x = 5{\cdot}8 \text{ or } 0{\cdot}2}$

(c) $y = x^2-6x+1$

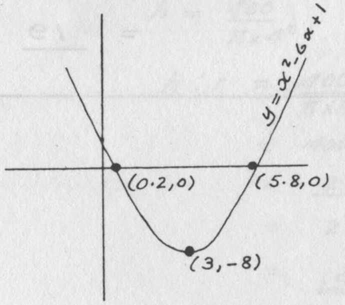
using results of part (b), $(5{\cdot}8,0)$ and $(0{\cdot}2,0)$ lie on the graph.

for max/min, $\dfrac{dy}{dx}=0$. ie $2x-6=0$

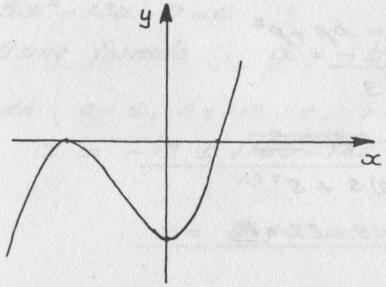
max/min when $x=3$ and $y=3^2-18+1=-8$

$\therefore\ (3,-8)$ lies on the graph

We have sufficient points to sketch the graph



(35)



$y = x^3+3x^2-4$.

(a) At P, $x=0$

$y = 0+0-4 = -4$

Coords of P are $(0,-4)$

(b) Q is point $(1,0)$ $\therefore\ x=1$ is a solution of the equation $x^3+3x^2-4=0$ and, thus, $(x-1)$ is a factor of x^3+3x^2-4

dividing x^3+3x^2-4 by $(x-1)$, we get

$$\begin{array}{r|l} & x^2+4x+4 \\ \hline x-1 & x^3+3x^2-4 \\ & \underline{x^3-x^2} \\ & 4x^2-4 \\ & \underline{4x^2-4x} \\ & 4x-4 \\ & \underline{4x-4} \end{array}$$

$\therefore\ x^2+4x+4$ is another factor

$\therefore\ x^2+4x+4=0$

$\therefore\ (x+2)^2=0$

$\therefore\ x=-2$

$\underline{R \text{ is the point } (-2,0)}$

(c) gradient $= \dfrac{dy}{dx} = 3x^2+6x$

for gradient $= 9$, $3x^2+6x=9$

$\therefore\ 3x^2+6x-9=0$

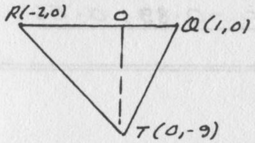
$\therefore\ x^2+2x-3=0$

$\therefore\ (x-1)(x+3)=0$

$\therefore\ x=1,-3$

$\underline{\text{gradient} = 9 \text{ at } (1,0) \text{ and } (-3,-4)}$

(d)



$RQ = 3$, $OT = 9$

$\underline{\text{Area } \Delta QTR = \tfrac{1}{2} \times 3 \times 9 = 13\tfrac{1}{2} \text{ sq units}}$

(36) $p = a - b$ and $q = bp + p^2$

(I) if $a = 2$, $b = -3$

then $\underline{p} = 2 - (-3) = \underline{5}$

and $q = (-3)5 + 5^2$

$\therefore \underline{q} = -15 + 25 = \underline{10}$

(II) $q = (bp + p^2)$

$\therefore q = b(a-b) + (a-b)^2$

$\therefore q = ab - b^2 + a^2 - 2ab + b^2$

$\therefore q = a^2 - ab$

$\therefore \underline{q = a(a-b)}$, as required

(III) for $q = \frac{1}{2}$, $b = -\frac{1}{3}$ then $q = a(a-b)$ can be written

$$\frac{1}{2} = a(a + \tfrac{1}{3})$$

multiplying by 6 $\quad 3 = 6a^2 + 2a$

$\therefore \underline{6a^2 + 2a - 3 = 0}$, as required

(IV) $6a^2 + 2a - 3 = 0$

Using formula "$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$"

for solution of "$ax^2 + bx + c = 0$"

we have :- $a = \dfrac{-2 \pm \sqrt{2^2 - 4(6)(-3)}}{12}$

$$a = \frac{-2 \pm \sqrt{4 + 72}}{12}$$

$$a = \frac{-2 \pm \sqrt{76}}{12}$$

$\therefore \underline{a = -0.89, 0.56}$

AEB. - Nov. 81 - B - A4

(37) (a) $5x^2 - 13x - 7 = 0$

Using formula :- $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

with $a = 5$, $b = -13$, $c = -7$

$\therefore x = \dfrac{-13 \pm \sqrt{169 - 140}}{10} = \dfrac{-13 \pm 17.58}{10}$

$\therefore \underline{x = 3.06 \text{ or } -0.46}$

(b) $\dfrac{m-12}{(m-3)(m+3)} + \dfrac{3}{2(m-3)}$

$$= \frac{2(m-12) + 3(m+3)}{2(m-3)(m+3)}$$

$$= \frac{2m - 24 + 3m + 9}{2(m-3)(m+3)}$$

$$= \frac{5m - 15}{2(m-3)(m+3)} = \frac{5(m-3)}{2(m-3)(m+3)}$$

$$= \underline{\frac{5}{2(m+3)}}$$

(c) $a = \dfrac{b-c}{b+c}$

(I) if $b = 17$, $c = 8$,

then $a = \dfrac{17-8}{17+8} = -\dfrac{9}{25}$

(II) $a(b+c) = b - c$

$\therefore ab + ac = b - c$

$\therefore ac + c = b - ab$

$\therefore c(a+1) = b(1-a)$

$\therefore C = \dfrac{b(1-a)}{a+1}$

A.E.B. _ Nov 1983 _ A _ I

(38) Car has a speed of x km/h.

(a) speed of lorry = $(x-30)$ km/h.

(b) time taken (by car) = $\frac{\text{distance}}{\text{speed}} = \frac{20}{x}$ hours.

(c) time taken (by lorry) = $\frac{20}{x-30}$ hours.

Time taken by car is $\frac{6}{60}$ hours less than time taken by lorry.

$$\therefore \frac{20}{x} = \frac{20}{x-30} - \frac{6}{60}$$

$$\therefore \frac{20}{x} = \frac{20}{x-30} - \frac{1}{10}$$

multiplying throughout by $10x(x-30)$

we have $200(x-30) = 200x - x(x-30)$

$$\therefore 200x - 6000 = 200x - x^2 + 30x$$

$$\therefore \underline{x^2 - 30x - 6000 = 0}\text{, as required}$$

Solving the above equation using the formula:-

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where:- $a = 1,\ b = -30,\ c = -6000$

$$\therefore x = \frac{30 \pm \sqrt{900 + 24000}}{2}$$

$$x = \frac{30 \pm \sqrt{24900}}{2}$$

$$\underline{x = 93{\cdot}9,\ -63{\cdot}9}$$

The speed of the lorry = $93{\cdot}9 - 30$

= $\underline{63{\cdot}9 \text{ km/h}}$

(39) (a) $N = 4(y - 36)$

old lowest mark = 36

new lowest mark (N_{36})

$N_{36} = 4(36-36)$

$\underline{N_{36} = 0}$

old highest mark = 61

new highest mark (N_{61})

$N_{61} = 4(61-36)$

$\underline{N_{61} = 100}$

Unchanged mark (N_u)

$N_u = 4(N_u - 36)$

$\therefore N_u = 4N_u - 144$

$\therefore 144 = 3N_u$

$\therefore \underline{N_u = 48}$

(b) $(2x+3)^2 = 20$

$\therefore 4x^2 + 12x + 9 = 20$

$\therefore \underline{4x^2 + 12x - 11 = 0}$, as required

To solve $4x^2 + 12x - 11 = 0$

we write $(2x+3)^2 = 20$

$\therefore 2x + 3 = \pm\sqrt{20}$

$\therefore 2x = -3 \pm \sqrt{20}$

$\therefore x = \frac{-3 \pm \sqrt{20}}{2}$

$\therefore \underline{x = 0{\cdot}74,\ -3{\cdot}74}$

(40) (a)

$$p - 2q = 3 \quad \text{——— ①}$$
$$pq = 2 \quad \text{——— ②}$$

from ② $p = \frac{2}{q}$

substituting in ① we have :- $\frac{2}{q} - 2q = 3$

$$\text{ie} \quad 2 - 2q^2 = 3q$$
$$2q^2 + 3q - 2 = 0$$
$$(2q-1)(q+2) = 0$$
$$\therefore 2q - 1 = 0 \quad \text{or} \quad q + 2 = 0$$
$$q = \tfrac{1}{2} \quad \text{or} \quad q = -2$$

subs. in ① $p - 1 = 3$ or $p + 4 = 3$

$$p = 4 \quad \text{or} \quad p = -1$$

∴ <u>Solutions</u> :- $\underline{p = -1,\ q = -2}$

or $\underline{p = 4,\ q = \tfrac{1}{2}}$

(b) $C = ax + bx^2$

substituting the given values for C and x :-

$$4 = 2a + 4b \quad \text{——— ③}$$
$$14 = 4a + 16b \quad \text{——— ④}$$

③×2 $\quad 8 = 4a + 8b$ ——— ⑤

④−⑤ $\quad 6 = 8b$

$$\therefore b = \frac{6}{8}$$

$\therefore \underline{b = \tfrac{3}{4}}$ subs. $b = \tfrac{3}{4}$ in ③

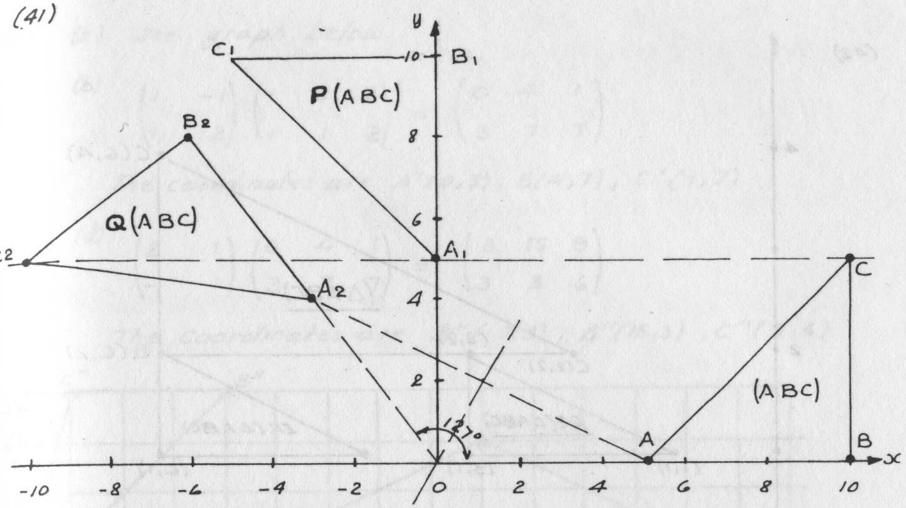
$$4 = 2a + 3$$
$$1 = 2a$$
$$\underline{\tfrac{1}{2} = a}$$

The formula can be written :- $C = \dfrac{x}{2} + \dfrac{3x^2}{4}$

$\therefore$ if $x = 6$

$$\therefore C = \frac{6}{2} + \frac{3 \times 36}{4}$$
$$= 3 + 27$$
$$= \underline{30 \text{ pence}}$$

(41)



(ii) <u>Coordinates of **P**(ABC)</u> :-

$$= \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 5 & 10 & 10 \\ 0 & 0 & 5 \end{pmatrix} = \begin{pmatrix} 0 & 0 & -5 \\ 5 & 10 & 10 \end{pmatrix}$$

<u>coordinates of **Q**(ABC)</u> :-

$$= -\frac{1}{5}\begin{pmatrix} 3 & 4 \\ -4 & 3 \end{pmatrix}\begin{pmatrix} 5 & 10 & 10 \\ 0 & 0 & 5 \end{pmatrix} = \begin{pmatrix} -3 & -6 & -10 \\ 4 & 8 & 5 \end{pmatrix}$$

The single transformation represented by **Q** is a rotation of approx (+ 127°), centre the origin (0,0). The angle is obtained by measurement and the centre by finding the point of intersection of the perpendicular bisectors of AA_2, BB_2, CC_2. (Note :- Angle of rotation $= \tan^{-1} \frac{-3}{4}$)

(iii) The inverse matrix P^{-1}

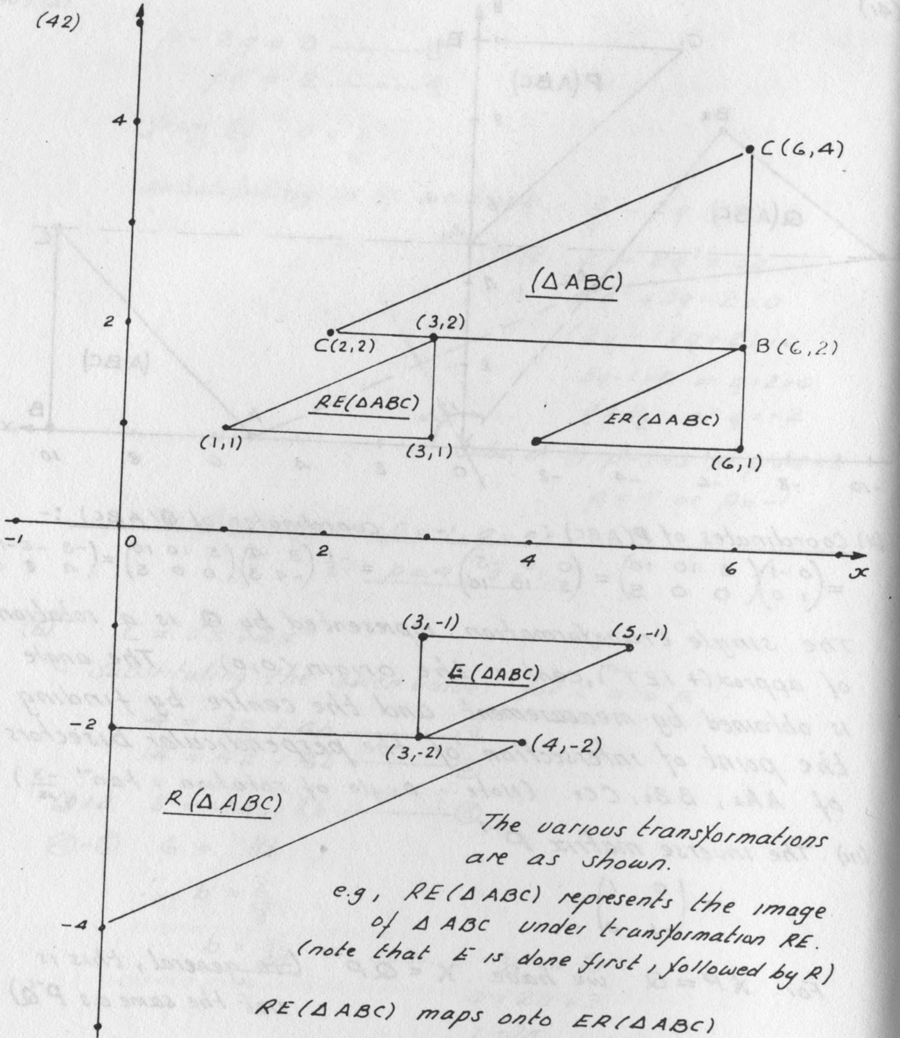
$$= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

For $XP = Q$ we have $X = QP^{-1}$ (in general, this is not the same as $P^{-1}Q$)

$$X = -\frac{1}{5}\begin{pmatrix} 3 & 4 \\ -4 & 3 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = -\frac{1}{5}\begin{pmatrix} -4 & 3 \\ -3 & -4 \end{pmatrix} = \underline{\underline{\frac{1}{5}\begin{pmatrix} 4 & -3 \\ 3 & 4 \end{pmatrix}}}$$

<u>X represents a rotation of $\tan^{-1} 4/3$</u> <u>= approx(+37°)</u>
<u>about origin (0,0)</u>

(42)



The various transformations are as shown.

e.g., RE(ΔABC) represents the image of ΔABC under transformation RE.
(note that E is done first, followed by R)

RE(ΔABC) maps onto ER(ΔABC) under the translation $\binom{3}{0}$

$\therefore$ T is the translation $\binom{3}{0}$

S maps (1,1) onto (2,2)
(3,1) onto (6,2)
(3,2) onto (6,4) $\Big\} \therefore$ S has matrix $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

S is enlargement, centre the origin, Scale factor = 2

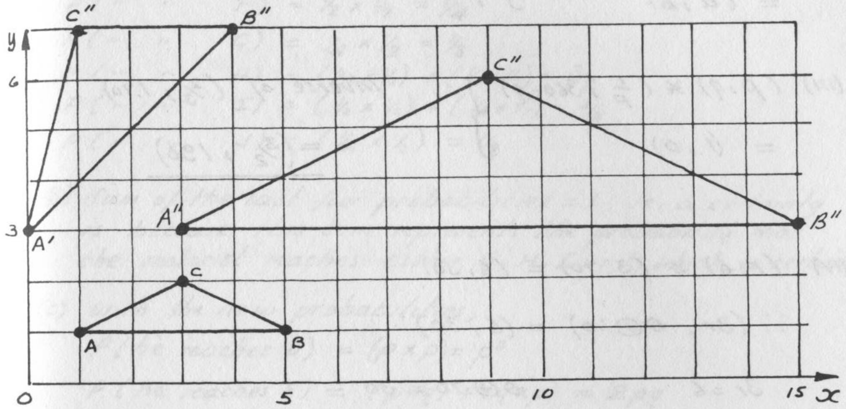
(43) (a) See graph below.

(b) $\begin{pmatrix} 1 & -1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 1 & 5 & 3 \\ 1 & 1 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 4 & 1 \\ 3 & 7 & 7 \end{pmatrix}$

The coordinates are A′(0,3), B′(4,7), C′(1,7)

(d) $\begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 0 & 4 & 1 \\ 3 & 7 & 7 \end{pmatrix} = \begin{pmatrix} 3 & 15 & 9 \\ 3 & 3 & 6 \end{pmatrix}$

The coordinates are A″(3,3), B″(15,3), C″(9,6)



Taking the area of ΔABC = 1 unit,
The area of ΔA′B′C′ = Determinant of $\begin{pmatrix} 1 & -1 \\ 1 & 2 \end{pmatrix} \times 1$

$= \{(1\times2)-(1\times-1)\} \times 1$

$= 3\times1 = 3$

The area of ΔA″B″C″ = Determinant of $\begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix} \times 3$

$= \{(2\times1)-(-1\times1)\} \times 3$

$= 3\times3 = 9$

Ratios of areas = 1 : 3 : 9

(The above could also, easily, be obtained by determining the areas of the triangles)

(44) for $S = \{(r,\theta) : r \geq 0 ,\ 0 \leq \theta < 360\}$

$(a*b)*(c*d) = (ac,\ b \oplus d)$ with $\oplus$ = addition modulo 360

(i) $(2,120) * (3,300) = \underline{(6,60)}$

(ii) $(a,b) * (1,0)$
$= (a,b)$

} Suggests $(1,0)$ is the identity element of S for operation $*$

(iii) $(p,q) * (\frac{1}{p},\ 360-q)$
$= (1,0)$

} Inverse of $(\frac{2}{3}, 170)$
$= \underline{(\frac{3}{2}, 190)}$

(iv) $(r,\theta) * (3,70) = (6,30)$

$\therefore (3r,\ \theta \oplus 70) = (6,30)$

$\therefore 3r = 6 \qquad \theta \oplus 70 = 30$

$\therefore \underline{r = 2} \qquad \therefore \theta = -40+0,\ -40+360,\ ----$

$\therefore \theta = -40,\ 320,\ ----$

$\underline{\theta = 320} \quad (0 \leq \theta < 360)$

(v) $[(r,\theta)*(r,\theta)] * (r,\theta) = (8,0)$

$\therefore (r^2,\ \theta \oplus \theta) * (r,\theta) = (8,0)$

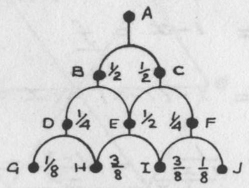
$\therefore (r^3,\ \theta \oplus \theta \oplus \theta) = (8,0)$

$\therefore r^3 = 8 \quad ,\ 3\theta \ (\text{mod. } 360) = 0$

$\therefore \underline{r = 2} \qquad \therefore 3\theta = 0,\ 360,\ 720,\ \ldots$

$\therefore \underline{\theta = 0,\ 120,\ 240.}$

(45) (a)



P(he reaches D) $= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

P(" " E) $= (\frac{1}{2} \times \frac{1}{2}) + (\frac{1}{2} \times \frac{1}{2}) = \frac{1}{2}$

P(" " F) $= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

P(" " G) $= \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$

P(" " H) $= (\frac{1}{4} \times \frac{1}{2}) + (\frac{1}{2} \times \frac{1}{2}) = \frac{3}{8}$

P(" " I) $= (\frac{1}{2} \times \frac{1}{2}) + (\frac{1}{4} \times \frac{1}{2}) = \frac{3}{8}$

P(" " J) $= (\frac{1}{4} \times \frac{1}{2}) = \frac{1}{8}$

(b) Sum of the last four probabilities = 1 ie, a certainty is because this sum represents the probability that the motorist reaches either G, H, I or J, which is certain.

(c) With the new probabilities,

P(he reaches D) $= (p \times p) = p^2$

P(he reaches E) $= (p \times q) + (q \times p) = 2pq$

P(he reaches F) $= (q \times q) = q^2$

(d) $(p+q)^2 = (p+q)(p+q) = p^2 + 2pq + q^2$
which is the same as the sum of the answers to part (c).

(46) (a) for the given functions,

$$\underline{fj} = 1 - \frac{1}{1-x} = \frac{1-x-1}{1-x} = \frac{-x}{1-x} = \frac{x}{x-1} = \underline{k}.$$

$$\underline{gj} = \frac{1}{\frac{1}{1-x}} = 1-x = \underline{f}$$

$$\underline{hj} = 1 - \frac{1}{\frac{1}{1-x}} = 1-(1-x) = x = \underline{i}$$

$$\underline{jj} = \frac{1}{1-\frac{1}{1-x}} = \frac{1}{\frac{1-x-1}{1-x}} = \frac{1-x}{-x} = 1 - \frac{1}{x} = \underline{h}$$

$$\underline{kj} = \frac{\frac{1}{1-x}}{\frac{1}{1-x} - 1} = \frac{1}{1-(1-x)} = \frac{1}{x} = \underline{g}$$

$$\underline{fk} = 1 - \frac{x}{x-1} = \frac{x-1-x}{x-1} = \frac{-1}{x-1} = \frac{1}{1-x} = \underline{j}$$

$$\underline{gk} = \frac{1}{\frac{x}{x-1}} = \frac{x-1}{x} = 1 - \frac{1}{x} = \underline{h}$$

$$\underline{hk} = 1 - \frac{1}{\frac{x}{x-1}} = 1 - \frac{x-1}{x} = \frac{x-x+1}{x} = \frac{1}{x} = \underline{g}$$

$$\underline{jk} = \frac{1}{1-\frac{x}{x-1}} = \frac{x-1}{x-1-x} = \frac{x-1}{-1} = 1-x = \underline{f}$$

$$\underline{kh} = \frac{\frac{x}{x-1}}{\frac{x}{x-1} - 1} = \frac{x}{x-x+1} = x = \underline{i}$$

	i	f	g	h	j	k
i	i	f	g	h	j	k
f	f	i	h	g	k	j
g	g	j	i	k	f	h
h	h	k	f	j	i	g
j	j	g	k	i	h	f
k	k	h	j	f	g	i

(b) identity function $= i$

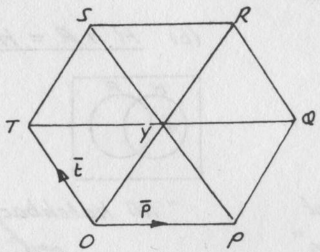
(c) inverse of $j = h$

inverse of $k = k$

(d) $(fg)h = hh = j$

$f(gh) = fk = j$

(47)



$$\overline{PT} = \overline{PO} + \overline{OT} = -\bar{p} + \bar{t} = \underline{\bar{t} - \bar{p}}$$

(a) $\overline{PS} = \overline{PY} + \overline{YS}$

PY is equal and parallel to OT

OY " " " " " OT

$\therefore \overline{PY} = \overline{YS} = \overline{OT} = \bar{t}$

$\therefore \underline{\overline{PS} = \overline{PY} + \overline{YS} = 2\bar{t}}$

(b) $\overline{OS} = \overline{OP} + \overline{PS} = \underline{\bar{p} + 2\bar{t}}$

$$\overline{OX} = \overline{OP} + \overline{PX} = \bar{p} + \tfrac{2}{3}(\overline{PT}) = \bar{p} + \tfrac{2}{3}(\bar{t} - \bar{p}) = \tfrac{1}{3}\bar{p} + \tfrac{2}{3}\bar{t} = \tfrac{1}{3}(\bar{p} + 2\bar{t}) = \tfrac{1}{3}\overline{OS}$$

$\therefore$ X lies on OS and $OX = \frac{OS}{3}$

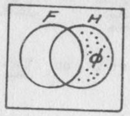
$$\frac{\overline{OX}}{\overline{XS}} = \frac{\overline{OX}}{\overline{XO} + \overline{OS}} = \frac{\frac{1}{3}\overline{OS}}{-\frac{1}{3}\overline{OS} + \overline{OS}}$$

$$\therefore \frac{OX}{XS} = \frac{1/3}{2/3}$$

$$\therefore \underline{\frac{OX}{XS} = \frac{1}{2}}$$

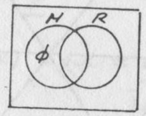
(48) For the sets defined,

(a) $F' \cap H = \phi$



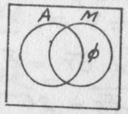
"Hatchbacks are not made in Great Britain"

(b) $H \cap R = H$



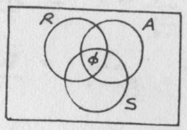
"All hatchbacks are red"

(c) $A \cup M = A$



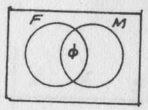
"All cars with engines greater than 2 litres capacity have automatic transmission"

(d)



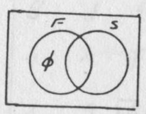
$R \cap A \cap S = \phi$

(e)



$F' \cap M = M$

(f)

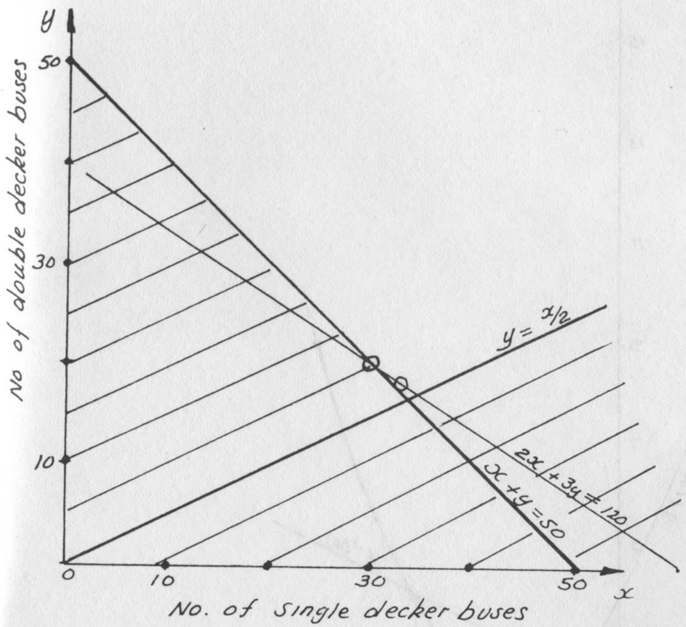


$F \cap S = F$

or $F \cup S = S$

or $F \subset S$

(49)



(i) The two inequalities are :- $y \geqslant \frac{x}{2}$ and $x + y \geqslant 50$

(ii) $40x + 60y = 2400$

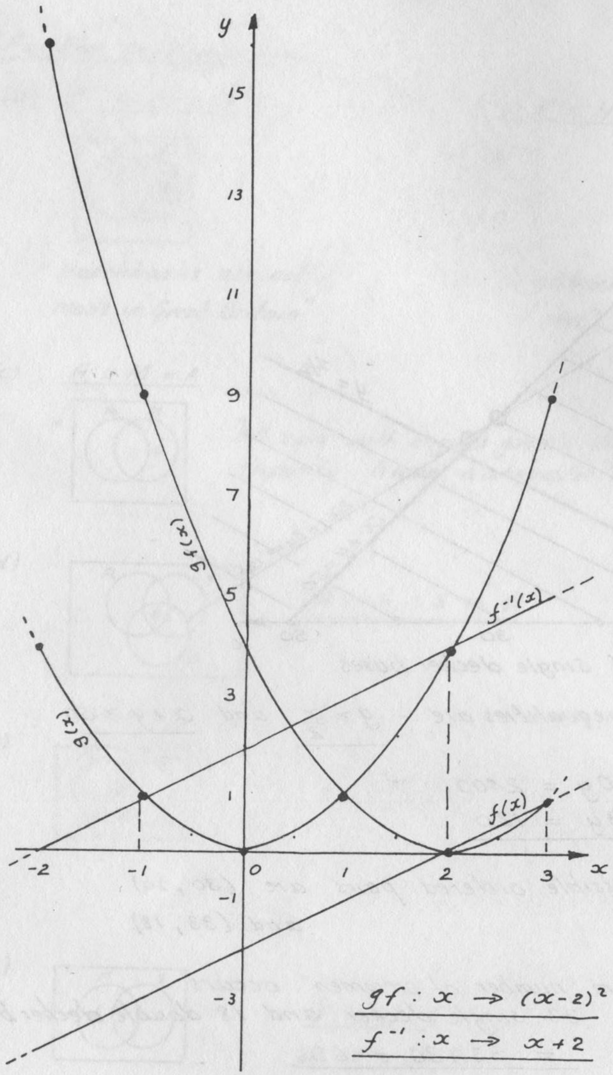
ie, $2x + 3y = 120$

(iii) The possible ordered pairs are (30, 20) and (33, 18)

(iv) Minimum number of crewmen occurs with 33 single decker and 18 double decker buses

$= 33 + 36 = 69$

(50)



$gf : x \to (x-2)^2$

$f^{-1} : x \to x+2$

from graph :-

(a) $gf(x) = f(x)$ when $x = 2, 3$

(b) $f^{-1}(x) = g(x)$ when $x = -1, 2$

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