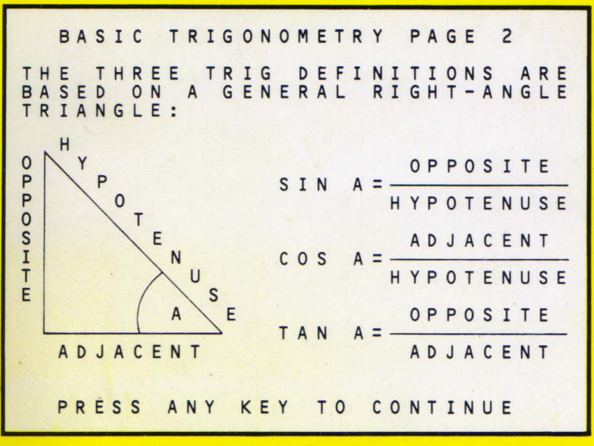
SHIELD SOFTWARE

MATHS 'O' LEVEL REVISION I

ALGEBRA AND TRIGONOMETRY



TOPICS COVERED

FRACTIONAL ALGEBRA
LINEAR EQUATIONS
SIMULTANEOUS EQUATIONS
QUADRATIC EQUATIONS
BASIC TRIGONOMETRY
VECTOR COMBINATION

CONTENTS

LOADING PROGRAMS

To load a program, reposition the tape to just before the correct program and type: SPECTRUM – LOAD“”
BBC B – CHAIN“”
CMB64 – LOAD“”
AMSTRAD – RUN“
MSX – LOAD“”,R

If an attempt to load is unsuccessful, make sure that you are starting from the correct place on the tape, check your cassette leads, and try adjusting the cassette volume control. If all else fails, remember that side 2 of each cassette is a copy of side 1.

BREAKING INTO PROGRAMS

Do not attempt to ‘break’ into any program unless you wish to remove it from memory. Any ‘break’ attempt will cause the programs to abort. Should this occur accidently, simply reset your computer as described in your manual and reload as if starting again.

ENDING A PROGRAM

To end a program, simply return to the first page (just keep selecting ‘Exit’ or ‘Return’ options) then select the ‘EXIT FROM PROGRAM’ option. This will clear out the current program and reset the computer ready for use.

THE PROGRAMS

Before running a program, it is recommended that you read the background notes for the relevant topics contained in this booklet. The topic notes give a brief précis of the program and a list of key facts relevant to that topic. Example questions from the self-test unit along with detailed answers are also given.

Coverage of each topic commences with a detailed introduction to the subject and the background theory. Having carefully studied this introduction, the student should proceed to the self-test unit, where exam-style questions can be attempted at any one of three levels. Students should start with the 'easier' level questions and should feel confident with them before proceeding through the 'medium' level to the 'harder' level.

Hints and detailed demonstration answers are available for any question asked in the self-test units. The demonstrations are done line-by-line to allow the student to follow the process at his own pace.

This topic covers the combination of fractions via any one of the four standard operations: Addition,Subtraction, Multiplication and Division. The method for each of these operations is examined in detail, with clear illustrations of each step in the appropriate process.

In order to enter your answers into the computer, you will need to use some symbol to represent the dividing line. The generally accepted convention is to use the 'slash' key. Instructions on how to enter this symbol are included in the section of the program entitled "Program Details".

KEY FACTS

A FRACTION is a number expressed as the ratio between two whole numbers.

The NUMERATOR is the 'top' number of a fraction. e.g. '2' in 2/3.

The DENOMINATOR is the bottom number of a fraction. e.g. '3' in 2/3.

The LOWEST COMMON MULTIPLE (LCM) of two numbers is the smallest number divisible by both numbers.

A PROPER fraction is one whose numerator is less than its denominator.

An IMPROPER fraction is one whose numerator is greater than its denominator.

QUESTION Given that X = 1/3 and Y = 1/4 calculate X + Y.

RECOMMENDED METHOD

STEP 1 Find a common denominator.

STEP 2 Convert the two fractions to contain this denominator.

STEP 3 Combine the two new fractions.

ANSWER

(STEP 1) The two denominators are 3 and 4. LCM of 3 & 4 is 12, so choose **12.**

(STEP 2) X = 1/3 = (4 x 1)/(4 x 3) = **4/12**
Y = 1/4 = (3 x 1)/(3 x 4) = **3/12**

(STEP 3) X + Y = (4/12) + (3/12) = (4 + 3)/12
= **7/12**

Answer: 7/12

QUESTION Given that X = 2/3 and Y = 4/5 calculate X ÷ Y.

RECOMMENDED METHOD

STEP 1 Invert the dividing fraction.

STEP 2 Multiply top by top and bottom by bottom.

ANSWER

(STEP 1) The divisor is 4/5 which inverts to **5/4**

(STEP 2) You now have (2/3) x (5/4)
= (2 x 5)/(3 x 4)
= 10/12
= **5/6**

Answer: 5/6

This topic covers the algebra of linear equations and their graphs. Particular attention is paid to the methods of finding the equation of a linear graph from its gradient and the co-ordinates of points through which it passes. These methods are analysed in detail through clear worked examples.

A demonstration is included for this topic, in which the student may choose values for 'M' and 'C' in the general equation of a straight line graph: Y = MX + C, and can then watch the graph being plotted on the screen.

These graphs can be plotted separately or superimposed on each other. This allows an accurate comparison to be made between similar graphs, thus allowing the student to see the effect of altering either of the coefficients in the general equation.

KEY FACTS

A LINEAR EQUATION is an equation in a single variable involving no squares or higher powers.

A LINEAR GRAPH is one consisting of a single straight line.

The GRADIENT of a linear graph is the increase in Y divided by the increase in X between any two points on the line.

All LINEAR equations give rise to LINEAR graphs.

All LINEAR graphs come from LINEAR equations.

All LINEAR graphs come from an equation of the form: Y = MX + C, where M is the value of the gradient.

QUESTION Find the equation of the straight line with gradient of 2, passing through the point (1,5).

RECOMMENDED METHOD

STEP 1 Use the general equation Y = MX + C.
STEP 2 Substitute the value of the gradient into the equation.
STEP 3 Substitute the values of X and Y at the given point into your equation.

ANSWER

(STEP 1) **Y = MX + C**
(STEP 2) Gradient = 2, so equation becomes **Y = 2X + C.**
(STEP 3) Graph passes through the point (1,5) so the equation must be satisfied when X = 1 and Y = 5.

i.e $5 = (2 \times 1) + C$
=> $5 = 2 + C$
=> **C = 3**
Answer: Y = 2X + 3

QUESTION Find the equation of the straight line passing through the points (1,3) and (2,4).

RECOMMENDED METHOD

STEP 1 Use the general equation Y = MX + C.
STEP 2 Substitute the values of X and Y at the first point into your equation.
STEP 3 Substitute the values of X and Y at the second point into your equation.
STEP 4 Solve the two resulting equations as for 'Simultaneous Equations'.

ANSWER

(STEP 1) **Y = MX + C**
(STEP 2) First point = (1,3), so equation must be satisfied when X = 1 and Y = 3,

i.e. $3 = (M \times 1) + C$
=> **M + C = 3 (i)**

(STEP 3) Second point = (2,4), so equation must be satisfied when X = 2 and Y = 4,

i.e. $4 = (M \times 2) + C$
=> **2M + C = 4 (ii)**

(STEP 4) Solving these as simultaneous equations gives M = 1 and C = 2.
So equation is **Y = 1X + 2**

This topic deals with quadratic equations, their graphs and their solution.

There are two main methods of solving a quadratic equation:

1) Solution by formula

IF $AX^2 + BX + C = 0$

THEN $X = \dfrac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

2) Solution by factorisation

IF the quadratic equation is rearranged to the form: $(X + A).(X + B) = 0$,

THEN $X = -A$ or $X = -B$.

The formula method is simply a question of memorising a formula and so can be easily dealt with outside this course. For this reason we will concentrate on the factorisation method.

A demonstration is included for this topic, in which the student may choose values for A, B and C in the general quadratic equation $Y = AX^2 + BX + C$, and can then watch the graph being plotted on the screen.

These graphs can be plotted separately or superimposed on each other. This offers an accurate comparison between similar graphs, thus allowing the student to see the effect of altering any of the equation's coefficients.

KEY FACTS

A QUADRATIC EQUATION in a variable X is one that can be re-arranged to the form:
$Y = AX^2 + BX + C$.

A FACTOR of a quadratic expression is any linear expression that divides it exactly.

FACTORISING a quadratic expression is the process of finding a pair of such factors.

QUESTION Factorise $X^2 - 3X + 2$

RECOMMENDED METHOD

STEP 1 Inspect the X^2 coefficient to find the two X parts.

STEP 2 Inspect the unitary coefficient to find the possible unit parts.

STEP 3 Inspect the X coefficient to find the correct option.

ANSWER

(STEP 1) The X^2 coefficient is 1 so the two X parts are X and X giving **(X).(X)**

(STEP 2) The unitary coefficient is +2, so possible unit parts are **(+2 and +1)** or **(−2 and −1)**

(STEP 3) The X coefficient is −3, so since the sum of −1 and −2 is −3, they are the correct unit parts.
so $\mathbf{X^2 - 3X + 2 = (X - 2).(X - 1)}$

QUESTION Solve $X^2 - 3X + 2 = 0$

RECOMMENDED METHOD

STEP 1 Factorise the quadratic expression.

STEP 2 Use the fact that if the product of two numbers equals zero, then one of the two numbers equals zero.

ANSWER

(STEP 1) As above $X^2 - 3X + 2 =$ **$(X - 2).(X - 1)$**

(STEP 2) If $X^2 - 3X + 2 = 0$ then $(X - 2)(X - 1) = 0$
Using the fact quoted above this implies either $X - 2 = 0$ or $X - 1 = 0$.
i.e. $X = 2$ or $X = 1$

This topic deals with simultaneous equations and examines how their solution relates to the point of intersection between their respective graphs.

There are two ways of solving simultaneous equations:

1. **Solution by algebraic manipulation**
   The two equations are re-arranged in such a way that they can be combined to eliminate one of the two variables. The value of the two variables is then easily found.

2. **Solution by graphical method**
   This method is highly inaccurate since it requires the drawing of two graphs and finding the point of intersection. It is rarely used but is sometimes referred to in questions.

Because of the inaccuracies in the graphical method, this course concentrates on the algebraic method.

KEY FACTS

A pair of SIMULTANEOUS EQUATIONS is a pair of equations in two variables that must be satisfied at the same time.

If you multiply or divide both sides of a valid equation by the same non-zero number then the resulting equation will also be valid.

If you add or subtract two valid equations then the resulting equation is also valid.

QUESTION Solve the following simultaneous equations:

$-X - 2Y = 3$

$2X - Y = 4$

RECOMMENDED METHOD

STEP 1 Find factors by which to multiply the two equations so that the coefficients in either X or Y become equal in size but opposite in sign.

STEP 2 Add the two new equations.

STEP 3 Find the value of the first variable from the resulting equation.

STEP 4 Find the value of the second variable by substituting into either of your equations the value of the first variable.

ANSWER

(STEP 1) Multiply the first equation by 2, this will bring the X coefficient to −2:

$2 \times (-X - 2Y = 3)$

$$\Rightarrow \mathbf{-2X - 4Y = 6}$$

Multiply the second equation by 1, leaving the X-coefficient as +2:

$$\mathbf{2X - Y = 4}$$

(STEP 2)

$$\begin{array}{r} -2X - 4Y = 6 \\ \underline{2X - Y = 4} \\ \mathbf{0X - 5Y = 10} \end{array}$$

(STEP 3) $-5Y = 10 \Rightarrow \mathbf{Y = -2}$

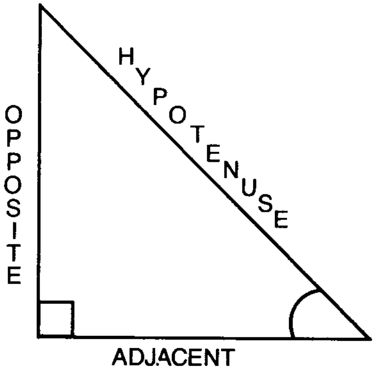
(STEP 4) Taking the second equation,

$2X - Y = 4 \Rightarrow 2X - (-2) = 4$

$\Rightarrow 2X + 2 = 4 \Rightarrow \mathbf{X = 1}$

This topic introduces the three trigonometric ratios of sine, cosine and tangent, along with Pythagoras' Theorem. The student is then shown how to use these equations to find the unknown lengths and internal angles of a right-angled triangle. Pythagoras' theorem relates the length of the three sides of any right-angled triangle, regardless of the size of the other two angles.

The basic definitions of sine, cosine and tangent are all derived from the same general diagram of a right-angled triangle:



KEY FACTS

The SINE of an angle is the ratio between the length of the side opposite the angle and the hypotenuse. [Opposite/Hypotenuse]

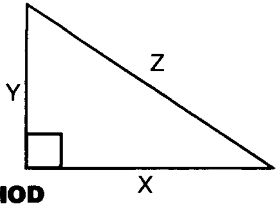
The COSINE of an angle is the ratio between the length of the side adjacent to the angle and the hypotenuse. [Adjacent/Hypotenuse]

The TANGENT of an angle is the ratio between the length of the side opposite and the length of the side adjacent. [Opposite/Adjacent]

PYTHAGORAS' THEOREM states that for any right angled triangle, the square of the hypotenuse length equals the square of the opposite length plus the square of the adjacent length.

1. FINDING THE THIRD SIDE

QUESTION Given X = 4 and Z = 5, calculate Y.



RECOMMENDED METHOD

STEP 1 Recall Pythagoras' Theorem

STEP 2 Substitute in the known values.

ANSWER

(STEP 1) Pythagoras' equation: $\mathbf{X^2 + Y^2 = Z^2}$

(STEP 2) X = 4 and Z = 5, so the equation becomes

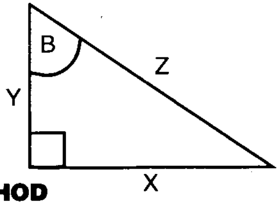
$$4^2 + Y^2 = 5^2$$

$=> Y^2 = 5^2 - 4^2 = 9$

$=>$ **Y = 3**

2. FINDING AN UNKNOWN ANGLE

QUESTION Given X = 4 and Z = 5, calculate angle B. (to nearest degree)



RECOMMENDED METHOD

STEP 1 Select the appropriate equation from sine, cosine and tangent.

STEP 2 Substitute in the known values.

ANSWER

(STEP 1) The known sides are X and Z which (relative to angle B) are the opposite and the hypotenuse. So correct choice is sine: **SIN (B) = OPPOSITE/HYPOTENUSE.**

(STEP 2) Opposite = X, so length of Opposite = 4
Hypotenuse = Z, so length of Hypotenuse = 5
so SIN(B) = 4/5 = 0.8
=> B = 53.13° or **B = 53°** (To nearest degree)

This topic deals with the combination of vectors both algebraically and geometrically. The operations covered are those of addition, subtraction, and multiplication by a scalar.

ALGEBRAIC VECTORS

To add or subtract two vectors, you simply add or subtract the equivalent entries.

e.g. If $\mathbf{A} = \begin{pmatrix}2\\4\end{pmatrix}$ & $\mathbf{B} = \begin{pmatrix}0\\3\end{pmatrix}$ then $\mathbf{A} + \mathbf{B} = \begin{pmatrix}2+0\\4+3\end{pmatrix} = \begin{pmatrix}2\\7\end{pmatrix}$

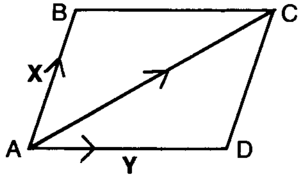
To multiply a vector by a scalar, you multiply each separate entry by the scalar.

e.g. If $\mathbf{A} = \begin{pmatrix}1\\3\end{pmatrix}$ then $2\mathbf{A} = \begin{pmatrix}2 \times 1\\2 \times 3\end{pmatrix} = \begin{pmatrix}2\\6\end{pmatrix}$

GEOMETRIC VECTORS

Vectors are described geometrically as being the displacement between two fixed points. They are combined via the parallelogram law.

e.g. if **X** = AB
and **Y** = AD
then **X** + **Y** = AB + AD
[Using AD = BC] = AB + BC
= **AD**



A demonstration is included in this topic in which the student may choose values for A and B in the general vector $\begin{pmatrix}A\\B\end{pmatrix}$ and can then see this vector plotted in various different ways on the screen

KEY FACTS

A VECTOR is a quantity that carries both magnitude and direction.

A UNIT VECTOR is a vector whose magnitude is 1.

A ZERO VECTOR is a vector whose magnitude is 0.

The POSITION VECTOR of a point P is the vector from the origin to that point. [=OP].

ALGEBRAIC QUESTION Given that $\mathbf{A} = \begin{pmatrix}1\\3\end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix}5\\0\end{pmatrix}$

calculate $2\mathbf{A} + \mathbf{B}$.

RECOMMENDED METHOD

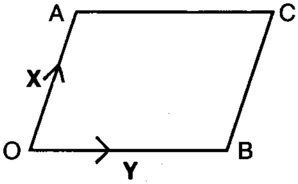
STEP 1 Combine the X coefficients.
STEP 2 Combine the Y coefficients.
STEP 3 Assemble the new vector.

ANSWER

(STEP 1) The X coefficient of 2**A** is (2 x 1) = 2
The X coefficient of **B** is 5.
=> X coefficient of 2**A** + **B** is 2 + 5 = **7.**
(STEP 2) The Y coefficient of 2**A** is (2 x 3) = 6
The Y coefficient **B** is 0
=> Y coefficient of 2**A** + **B** is 6 + 0 = **6.**
(STEP 3) Assembling gives $2\mathbf{A} + \mathbf{B} = \begin{pmatrix}7\\6\end{pmatrix}$

GEOMETRIC QUESTION

Given that the following diagram is a parallelogram and that vector **X** = OA and vector **Y** = OB, describe the vector **X** + **Y**.



RECOMMENDED METHOD

STEP 1 Find a suitable starting point.
STEP 2 Trace the first part of the vector.
STEP 3 Trace the second part of the vector.
STEP 4 Combine the two parts to make the whole.

ANSWER

(STEP 1) The best starting point is O.
(STEP 2) The first part is **X** which is OA.
(STEP 3) The second part is +**Y** which is OB, or, since OACB is a parallelogram, AC.
(STEP 4) **OA + AC = OC.**

Answer: **X + Y = OC**

This package comprises Part I of a two-part revision system. Part II is also available through your stockist, and covers the following topics:

Matrices
Shapes and Solids
Venn Diagrams
Probability
Chart Representation
Percentages and Averages

Also available from Shield Software is the O-Level Examiner, a multiple choice program capable of setting over 1,000,000 different exams. Each exam is set according to syllabus information entered by the student, thus ensuring maximum compatibility with the student's needs. Each exam ends with a full graded result plus a breakdown of the performance in the key areas of the syllabus.